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Stochastic Models in Robotics

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Outline of this talk

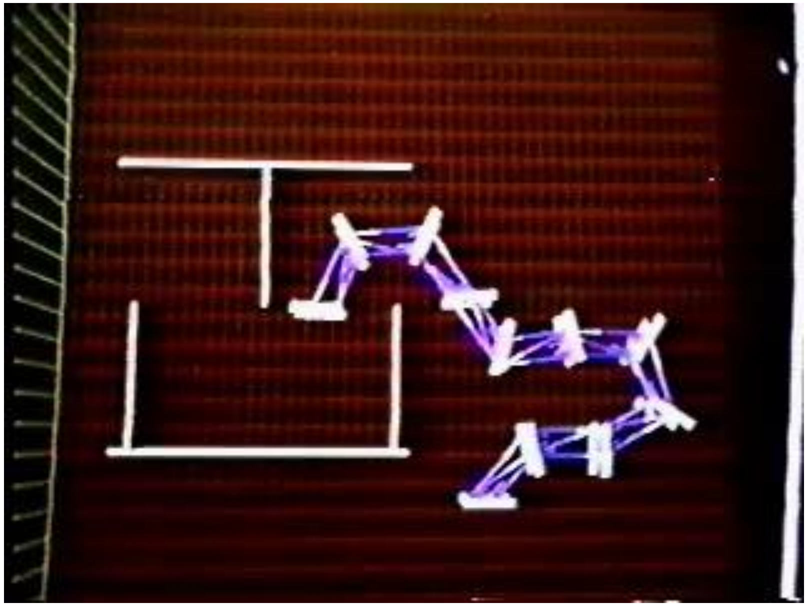
- From Snake-Robot Motion Planning to Uncertainty Propagation in Nonholonomic Vehicles
- Spherical Motors and Medical Image Registration
- Hand-Eye Calibration and Ultrasound
- Robotic Diagnosis, Repair, and Replication
- Acknowledgements: W. Park,Y. Zhou, M.K. Kim, R. Jernigan,
J. Burdick, I. Ebert-Uphoff, A. Okamura, N. Cowan, S. Lee,
M. Moses, M. Kobilarov, …

Topic 1:

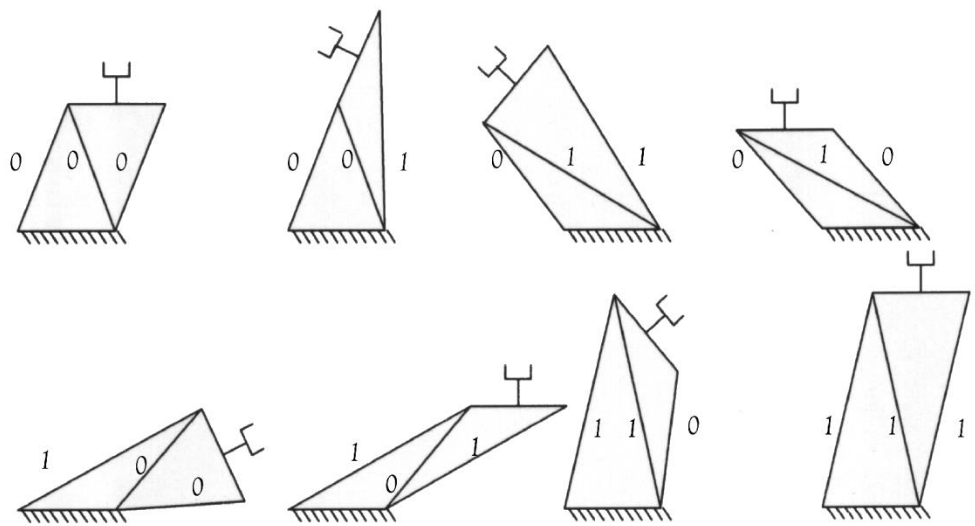
From Snake Robots to Uncertainty Propagation in Vehicles

Hardware from the PhD Years

Simulations from the PhD Years



A Binary Manipulator with One Module

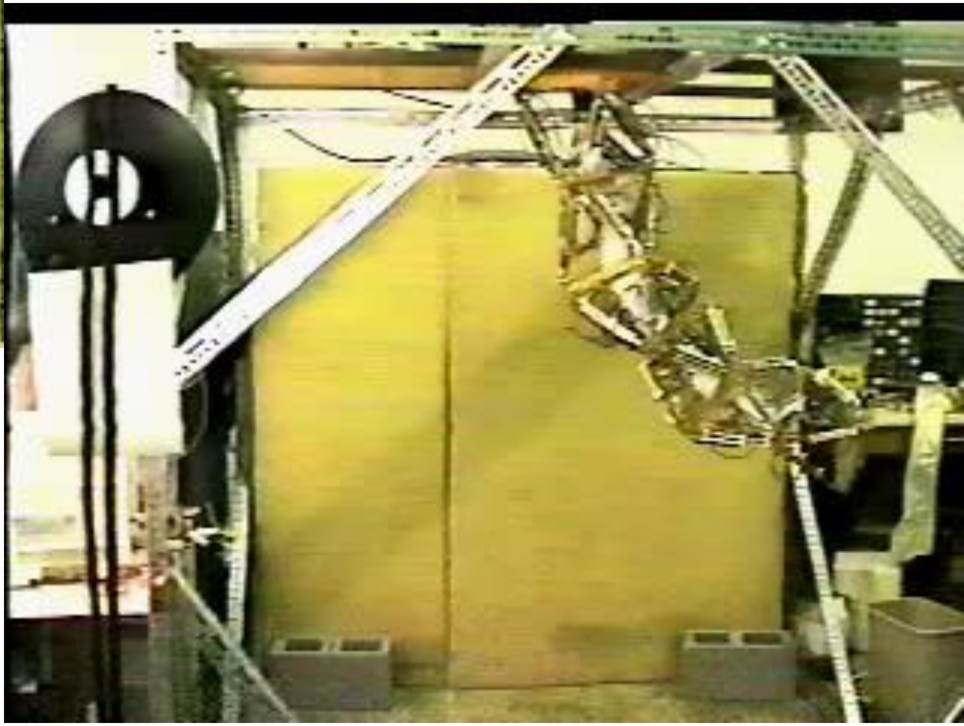


0: retracted state; 1: extended state

Examples of Binary Manipulators

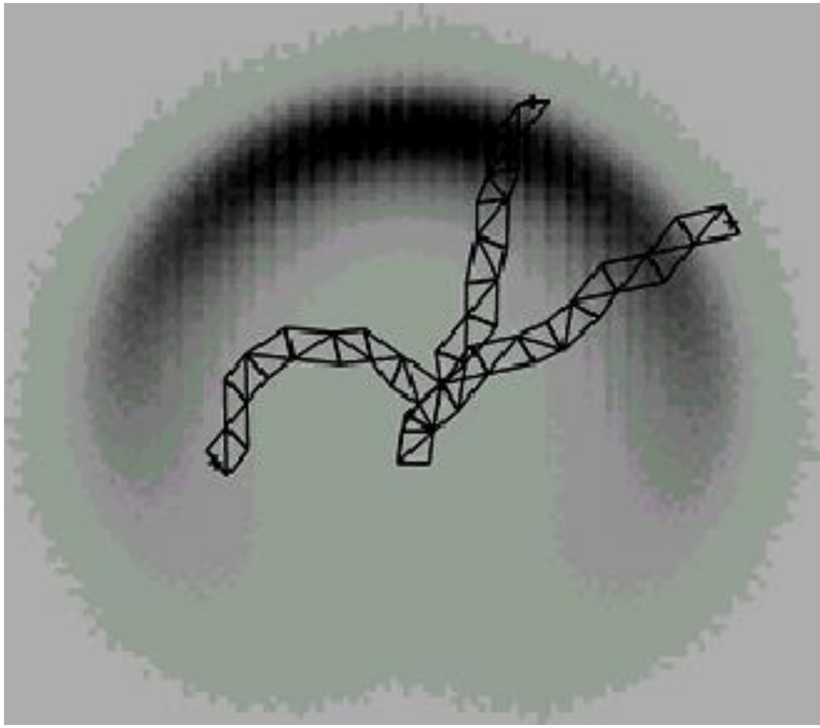
Imme Ebert-Uphoff, CSU

David Stein, VP Siemens



Workspace Density

- It describes the density of the reachable frames in the work space.
- It is a probabilistic measure-ment of accuracy over the workspace.



Rigid-Body Motions in Euclidean Space

$$SE(n) = (\mathbb{R}^n, +) \rtimes SO(n)$$

$$g_1 \circ g_2 = (R_1, \mathbf{t}_1) \circ (R_2, \mathbf{t}_2) = (R_1 R_2, R_1 \mathbf{t}_2 + \mathbf{t}_1)$$

$$g^{-1} = (R^T, -R^T \mathbf{t}) \quad \text{and} \quad e = (\mathbb{I}, \mathbf{0})$$

Rigid-Body Motion Group

- **Special Euclidean motion group *SE(N)***
  - An element of G=*SE(N)*:

$$g = \begin{pmatrix} A & a \\ 0^T & 1 \end{pmatrix}$$

  - Group operation: matrix multiplication
- **For example, an element of *SE(2)* in polar coordinates:**

$$g(\phi, r, \theta) = \begin{pmatrix} \cos\phi & -\sin\phi & r\cos\theta \\ \sin\phi & \cos\phi & r\sin\theta \\ 0 & 0 & 1 \end{pmatrix}$$

Exponential Coordinates for SE(2)

$$
\begin{aligned}
g(v_1, v_2, \alpha) &= \exp(X) \\
&= \begin{pmatrix} \cos\alpha & -\sin\alpha & t_1 \\ \sin\alpha & \cos\alpha & t_2 \\ 0 & 0 & 1 \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
t_1 &= [v_2(-1 + \cos\alpha) + v_1 \sin\alpha]/\alpha \\
t_2 &= [v_1(1 - \cos\alpha) + v_2 \sin\alpha]/\alpha.
\end{aligned}
$$

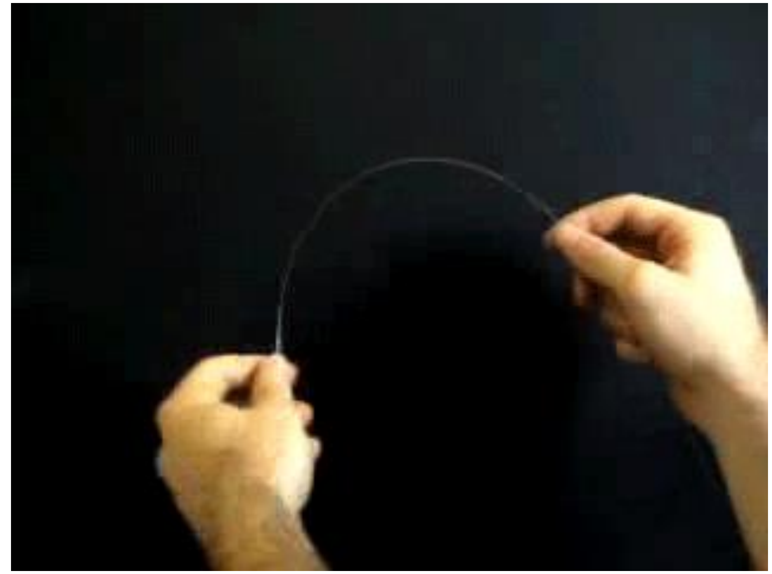
Convolution and the SE(3) Fourier Transform

$$(f_1 * f_2)(g) = \int_G f_1(h) f_2(h^{-1} \circ g) dh$$

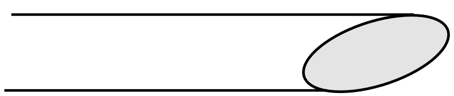
$$F(f_1 * f_2) = F(f_2) F(f_1)$$

Chirikjian, G.S., Kyatkin, A.B., Engineering Applications of Noncommutative Harmonic Analysis, CRC Press, 2001.

Flexible Needles with Bevel tip



http://research.vuse.vanderbilt.edu/MEDLab/research_files/needlesteer.htm



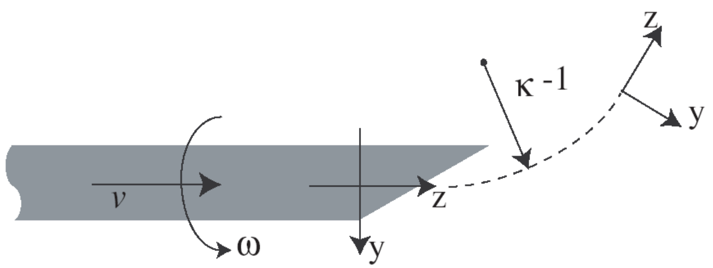
Needle with a bevel tip

Needle Model

- Deterministic nonholonomic model

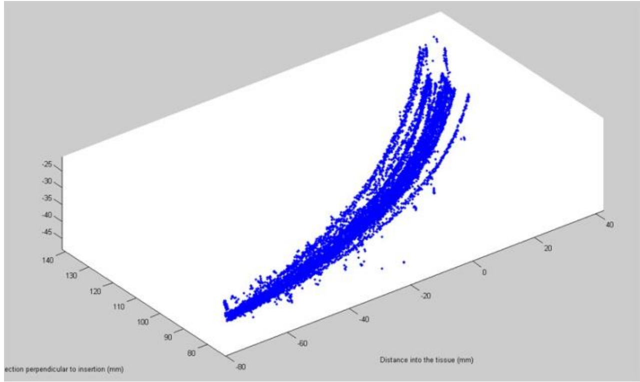
$$\xi = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} \kappa v \\ 0 \\ \omega \\ 0 \\ 0 \\ v \end{bmatrix}$$

v : insertion speed ω : twisting angular velocity



W. Park, J.S. Kim, Y. Zhou, N. Cowan, A. Okamura, G. Chirikjian, ``Diffusion-based motion planning for a nonholonomic flexible steerable needle model,” ICRA 2005

R. J. Webster III, J. S. Kim, N. J. Cowan, G. S. Chirikjian and A. M. Okamura, "Nonholonomic Modeling of Needle Steering," International Journal of Robotics Research, Vol. 25, No. 5-6, pp. 509-525, May-June 2006.



Experiments by Dr. Kyle Reed

Stochastic needle model

- **Stochastic model**
  - The effect of noise is included.
  - The noise terms are included in the inputs.

$$\omega(t) = \lambda_1 w_1(t) \qquad w_i(t) \text{ : Unit Gaussian white noise}$$

$$v(t) = 1 + \lambda_2 w_2(t) \qquad \lambda_i \text{ : Strength of noise}$$

$$(g^{-1}\dot{g})^{\vee} dt = \begin{bmatrix} \kappa & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}} dt + \begin{bmatrix} 0 & 0 & \lambda_1 & 0 & 0 & 0 \\ \kappa\lambda_2 & 0 & 0 & 0 & 0 & \lambda_2 \end{bmatrix}^{T} \begin{bmatrix} dW_1 \\ dW_2 \end{bmatrix}$$

g : SE(3) frame for needle tip pose

$W_i(t)$: Wiener process

Y. Zhou and G. Chirikjian, ``Probabilistic Models of Dead-Reckoning Error in Nonholonomic Mobile Robots." ICRA 2003.

W. Park, Y. Liu, Y. Zhou, M. Moses, G. S. Chirikjian. ``Kinematic state estimation and motion planning for stochastic nonholonomic systems using the exponential map," Robotica. 26(4):419–434. 2008

Stochastic Vehicle Models

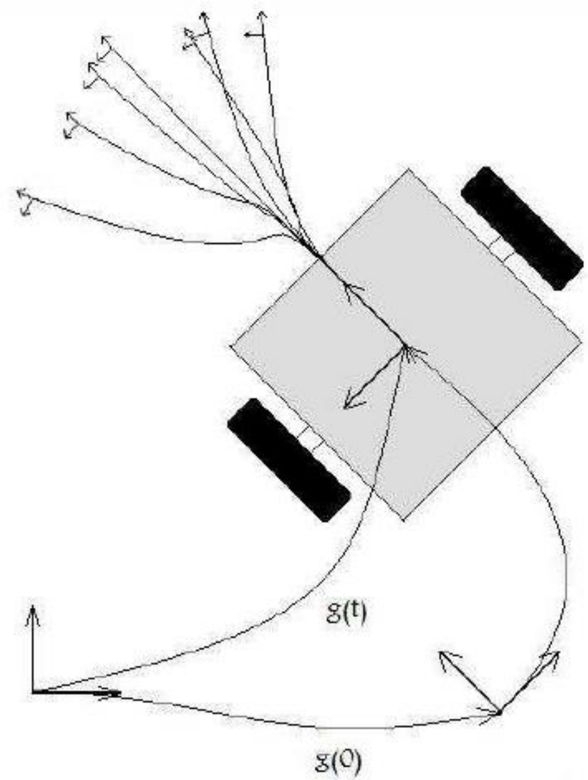
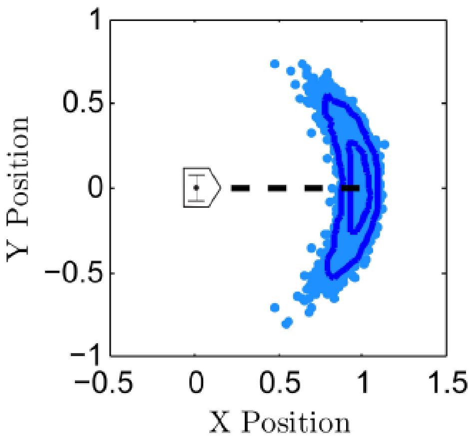


Fig. 0.1. A Kinematic Cart with an Uncertain Future Position and Orientation

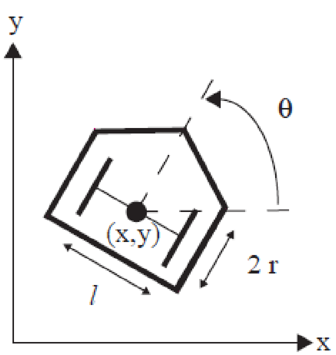
$$d\phi_1 = \omega(t)dt + \sqrt{D}dw_1$$
$$d\phi_2 = \omega(t)dt + \sqrt{D}dw_2$$

SDE for the Kinematic Cart

(Zhou and Chirikjian, ICRA 2003)



(a)



(b)

$$\begin{pmatrix} dx \\ dy \\ d\theta \end{pmatrix} = \begin{pmatrix} \frac{r}{2}(\omega_1 + \omega_2)\cos\theta \\ \frac{r}{2}(\omega_1 + \omega_2)\sin\theta \\ \frac{r}{\ell}(\omega_1 - \omega_2) \end{pmatrix} dt + \sqrt{D} \begin{pmatrix} \frac{r}{2}\cos\theta & \frac{r}{2}\cos\theta \\ \frac{r}{2}\sin\theta & \frac{r}{2}\sin\theta \\ \frac{r}{\ell} & -\frac{r}{\ell} \end{pmatrix} \begin{pmatrix} dw_1 \\ dw_2 \end{pmatrix}$$

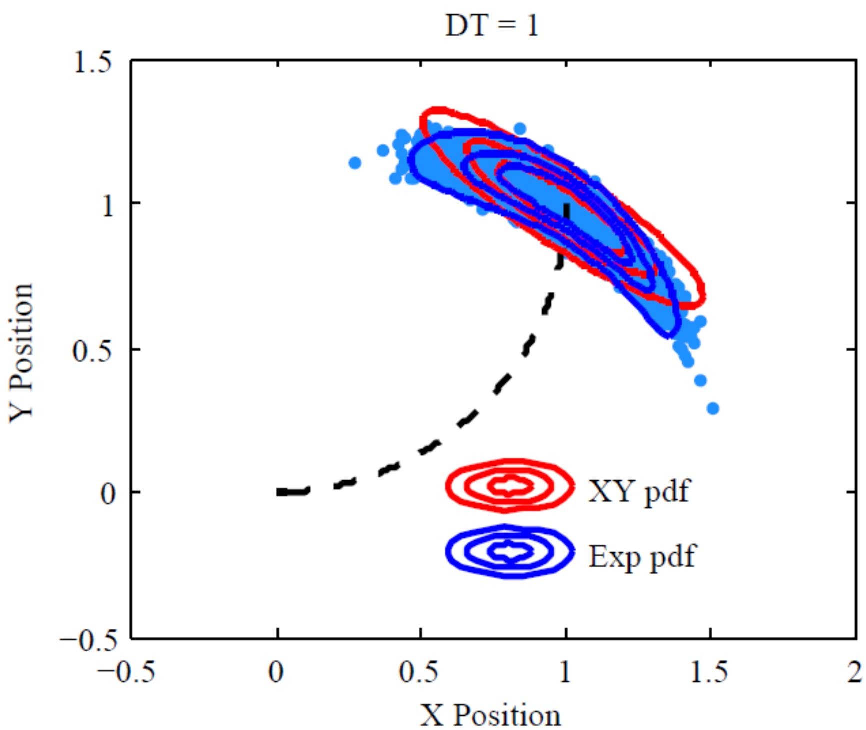
$$
\begin{pmatrix} dx \\ \\ dy \\ \\ d\theta \end{pmatrix} = \begin{pmatrix} r\omega\cos\theta \\ \\ r\omega\sin\theta \\ \\ 0 \end{pmatrix} dt + \sqrt{D} \begin{pmatrix} \frac{r}{2}\cos\theta & \frac{r}{2}\cos\theta \\ \\ \frac{r}{2}\sin\theta & \frac{r}{2}\sin\theta \\ \\ \frac{r}{L} & -\frac{r}{L} \end{pmatrix} \begin{pmatrix} dw_1 \\ \\ dw_2 \end{pmatrix} \tag{0.4}
$$

Corresponding to an SDE is a Fokker-Planck equation

$$
\frac{\partial f}{\partial t} = -r\omega\cos\theta\frac{\partial f}{\partial x} - r\omega\sin\theta\frac{\partial f}{\partial y} +
$$
$$
\frac{D}{2}\left(\frac{r^2}{2}\cos^2\theta\frac{\partial^2 f}{\partial x^2} + \frac{r^2}{2}\sin 2\theta\frac{\partial^2 f}{\partial x\partial y} + \frac{r^2}{2}\sin^2\theta\frac{\partial^2 f}{\partial y^2} + \frac{2r^2}{L^2}\frac{\partial^2 f}{\partial\theta^2}\right).
$$

There is a very clean coordinate-free way of writing these SDEs and FPEs. Namely,

$$
\left(g^{-1}\frac{dg}{dt}\right)^{\vee} dt = r\omega\mathbf{e}_1 dt + \frac{r\sqrt{D}}{2}\begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 2/L & -2/L \end{pmatrix} d\mathbf{w}
$$



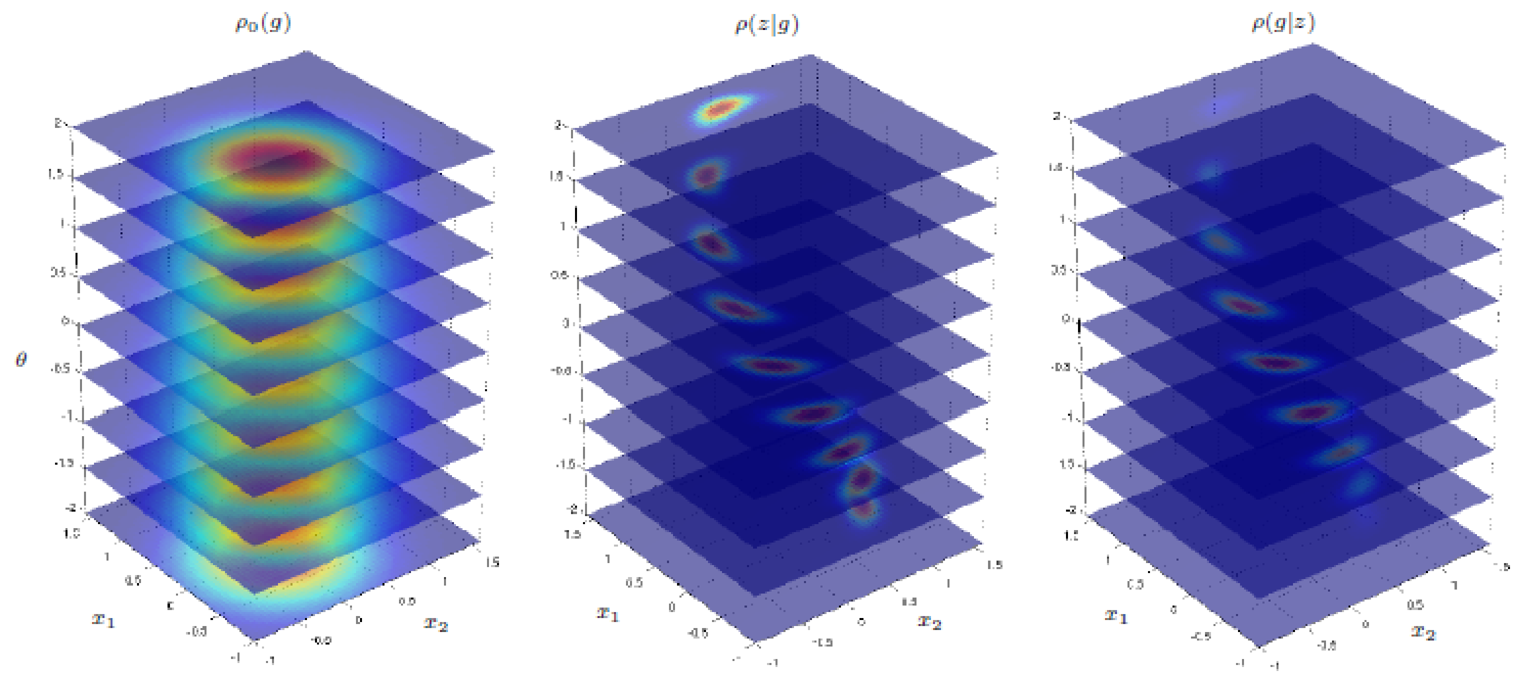
A. Long, K. Wolfe, M. Mashner, G. Chirikjian, ``The Banana Distribution is Gaussian” RSS 2012

$$\int_G \log^\vee\left(\mu^{-1}\circ g\right) f(g)dg = \mathbf{0} \qquad \Sigma = \int_G \log^\vee(\mu^{-1}\circ g)[\log^\vee(\mu^{-1}\circ g)]^T f(g)dg$$

$$f(g;\mu,\Sigma) = \frac{1}{c(\Sigma)}\exp\left[-\frac{1}{2}\mathbf{y}^T\Sigma^{-1}\mathbf{y}\right] \qquad \mathbf{y} = \log(\mu^{-1}\circ g)^\vee$$

(Work with Marin Kobilarov – CDC'14)

Distribution update using range-bearing measurements



a) prior $\rho_0(g)$ b) meas. model $\rho(z|g)$ c) combined density $\rho(g|z)$

- Note: all densities above displayed in $q = (x_1, x_2, \theta)$ for clarity
- density $\rho(g|z)$ is the full (non-parametric) nonlinear (and non-Gaussian in pose space) density that we aim to approximate.

Topic 2:

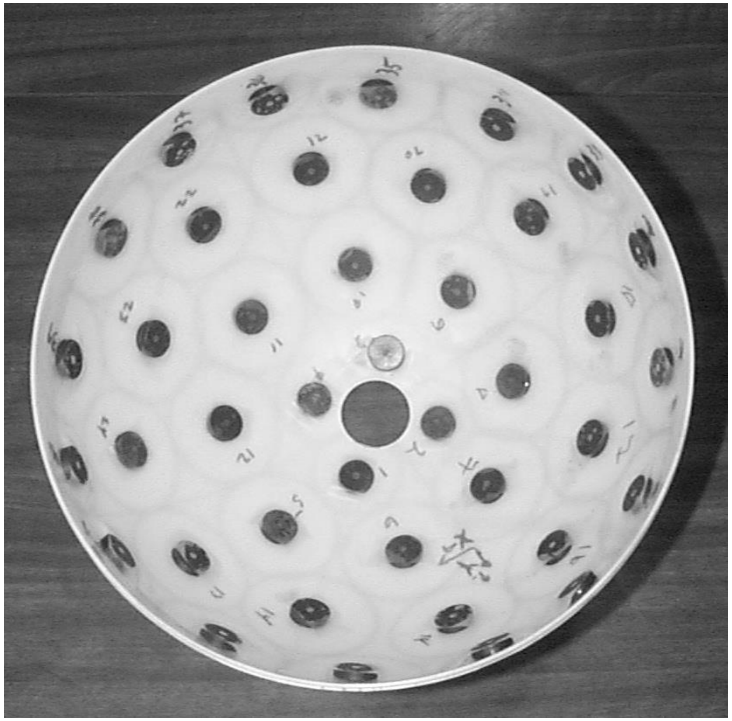
Spherical Motors and Medical Image Registration

The Spherical Motor/Encoder

With David Stein and Ed Scheinerman

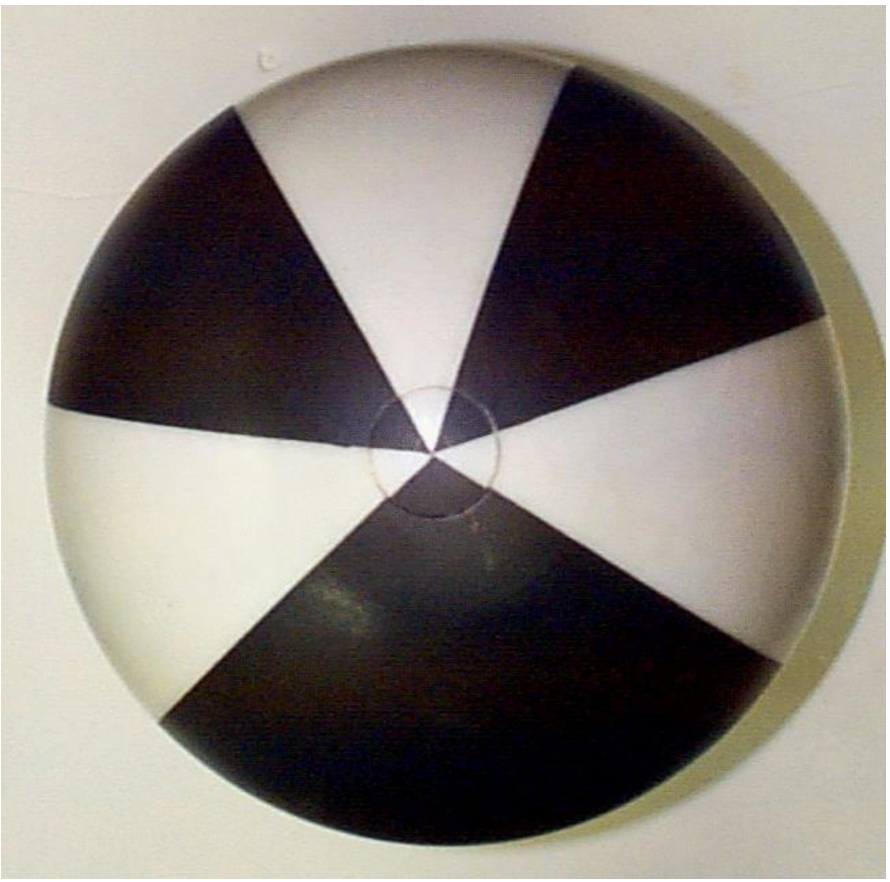
Our Prototype Motor

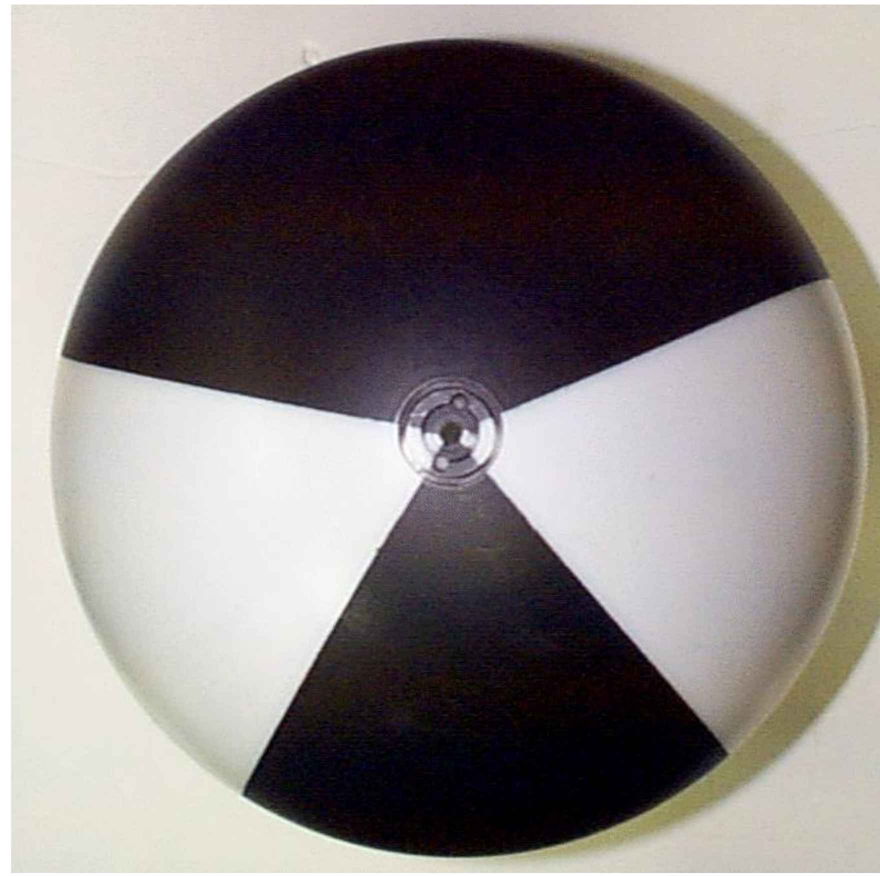
Stator



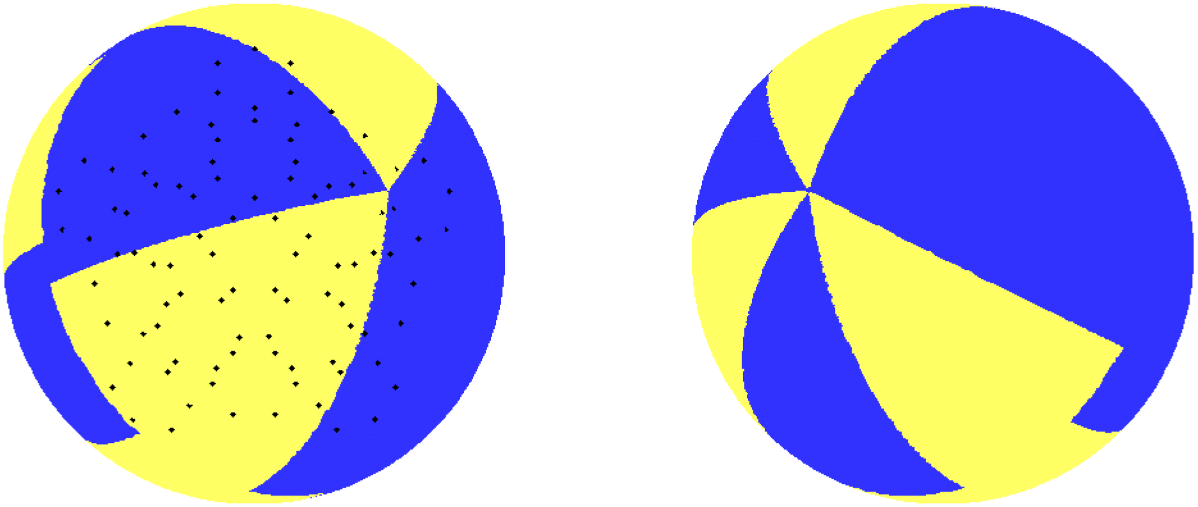
Rotor

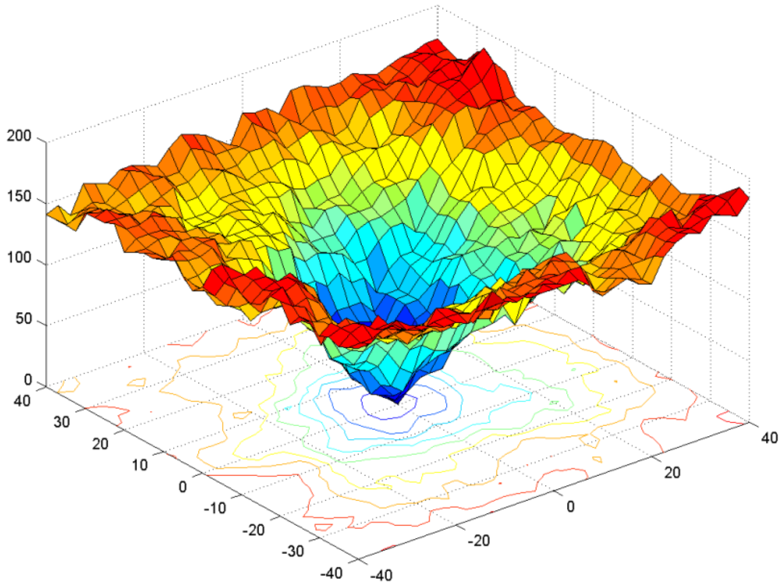
The Encoder



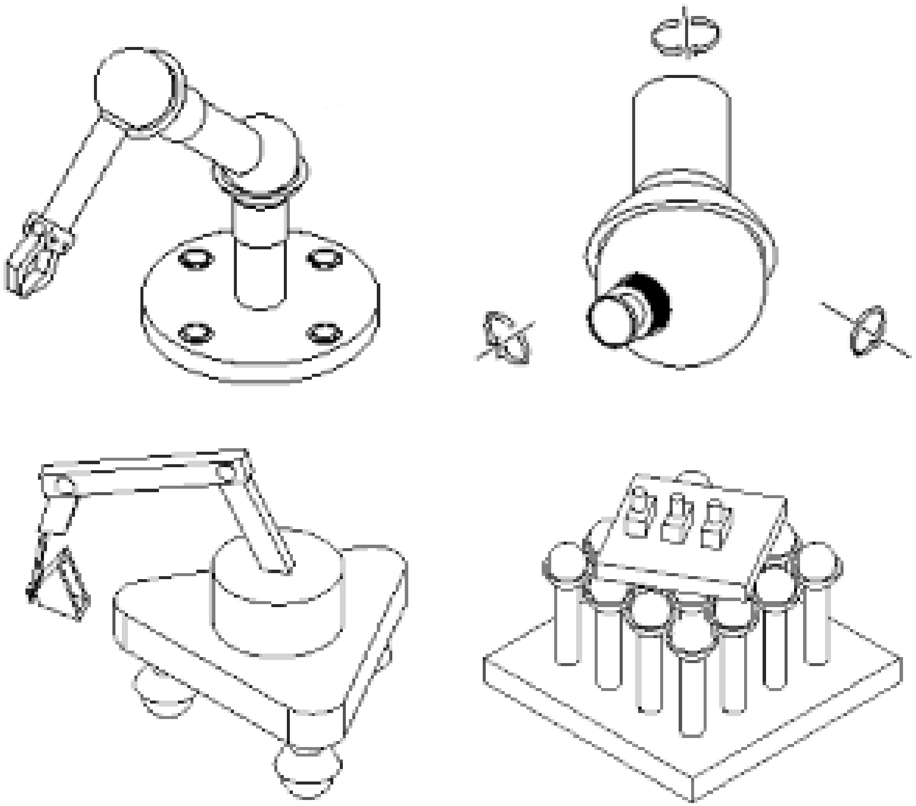


Operating Principles of the Spherical Encoder





Potential Applications

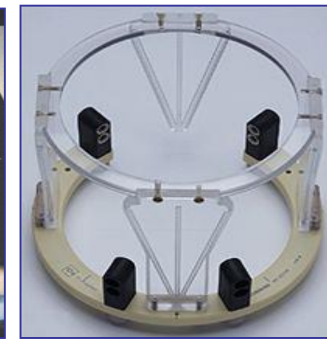


Novel Algorithms for Robust Registration of Fiducials in CT and MRI

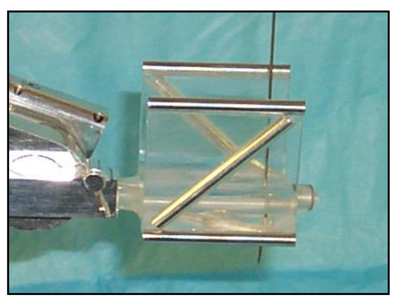
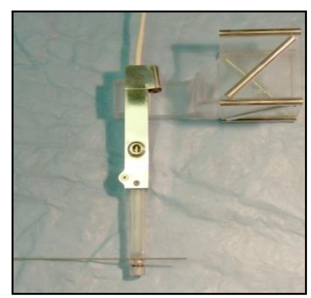
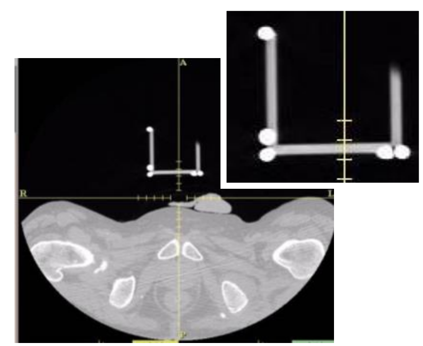
With Sangyoon Lee, Gabor Fichtinger, Attila Tanacs

CRW CT **CRW MRI** **Kelly CT** **Leibinger CT**



Some of the frequently used stereotactic fiducial frames

Stereotactic localizers mounted on robotic needle drivers

Incomplete data

References

1) G. S. Chirikjian, Stochastic Models, Information Theory, and Lie Groups, Vol. 1, 2, Birkhauser, 2009,2011.
2) G.S. Chirikjian, A.B. Kyatkin, Engineering Applications of Noncommutative Harmonic Analysis, CRC Press, 2001.
3) Jain, A., Zhou, Y., Mustufa, T., Burdette, E.C., Chirikjian, G.S ., Fichtinger, G., ``Matching and Reconstruction of Brachytherapy Seeds using the Hungarian Algorithm (MARSHAL),'' Medical Physics 32 (11): 3475-3492 Nov 2005
4) Jain, A., Mustafa, T., Zhou, Y., Burdette, E.C ., Chirikjian, G.S ., Fichtinger, G., ``Robust fluoroscope tracking fiducial,'' Medical Physics 32 (10): 3185-3198, Oct 2005
4) Stein, D., Scheinerman, E.R., Chirikjian, G.S., ``Mathematical models of binary spherical-motion encoders, IEEE-ASME Trans. on Mechatronics 8 (2): 234-244, 2003

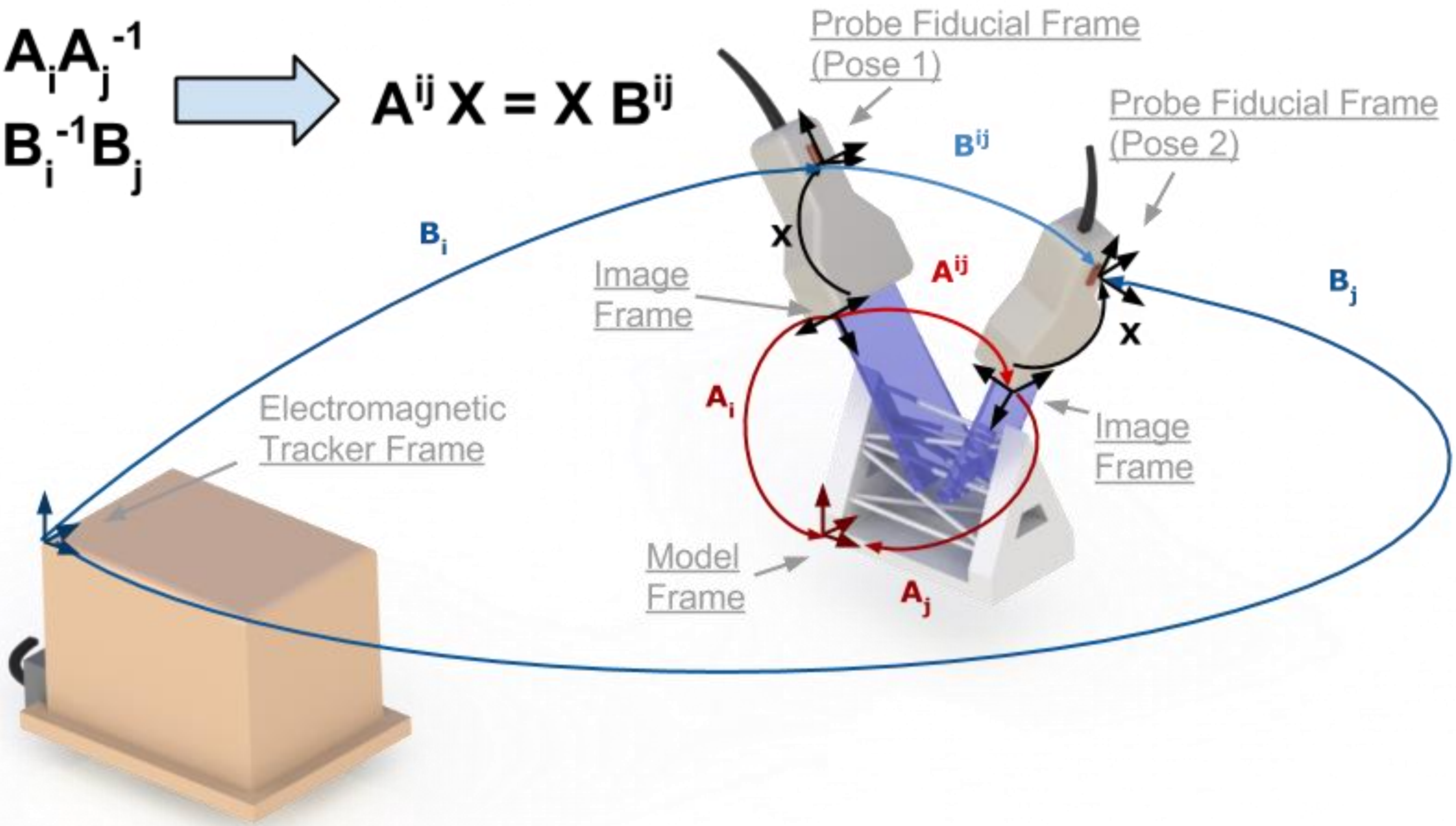
Topic 3:

Hand-Eye Calibration and Ultrasound Sensor Calibration

Ultrasound Calibration

(with E. Boctor, M.K. Ackerman, A. Cheng)

$$A^{ij} = A_i A_j^{-1}$$
$$B^{ij} = B_i^{-1} B_j$$
$$\Rightarrow \quad A^{ij} X = X B^{ij}$$



Definitions

- $$AX=XB$$

$A, B, X \in SE(3)$ where $\mathrm{SE}(3) = \mathbb{R}^3 \ltimes SO(3)$ and
$\mathrm{SO}(3) \coloneqq \{R \in SO(3) | RR^T = \mathbb{I}, \det(R) = +1\}$

SE(3) is the Lie Group describing rigid body motions in 3-dimensional space, i.e.:

H $\in SE(3)$, where $H(R, \mathbf{t}) = \begin{pmatrix} R & \mathbf{t} \\ \mathbf{0}^T & 1 \end{pmatrix}$ and

R $\in SO(3)$ (a proper rotation matrix),
t $\in \mathbb{R}^3$ (a translation vector)

Lie Groups and Rigid Body Motion

From Screw theory we can write:

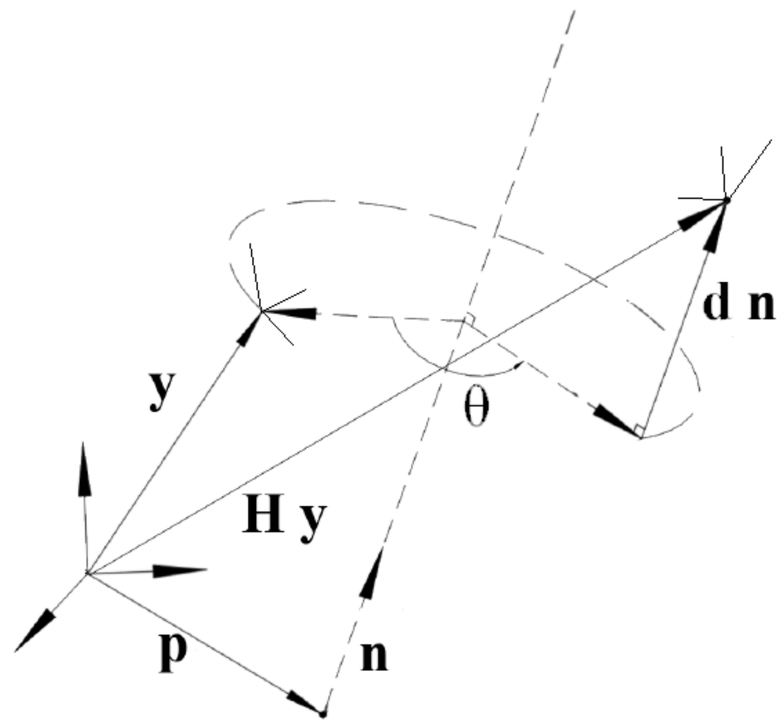
$$H = \begin{pmatrix} e^{\theta N} & (\mathbb{I}_3 - e^{\theta N})\mathbf{p} + d\,\mathbf{n} \\ \mathbf{0}^T & 1 \end{pmatrix}$$

where

$$N = \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{pmatrix}$$

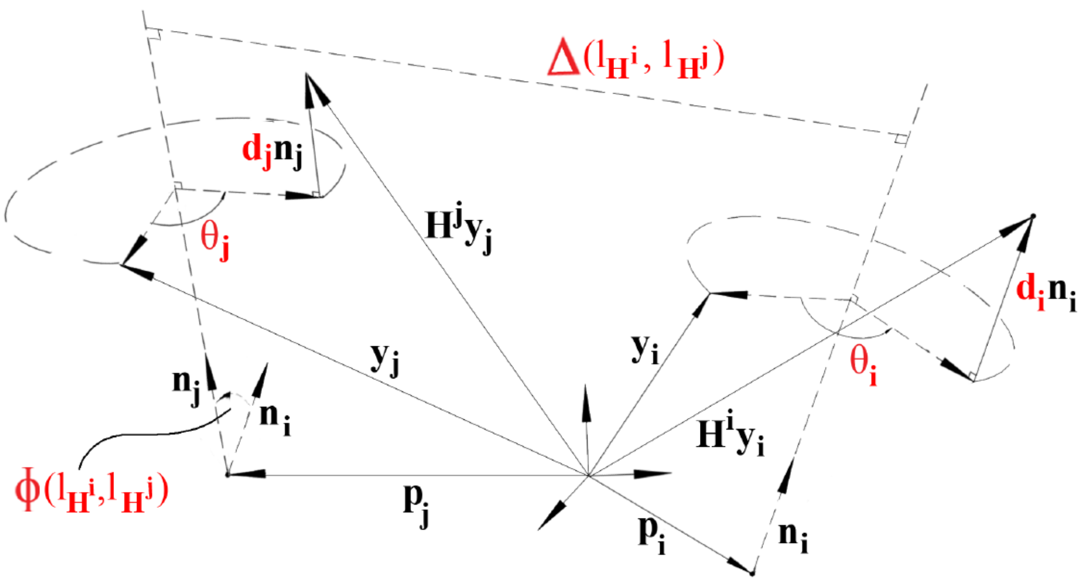
$$\mathbf{n} = [\mathbf{n_1}, \mathbf{n_2}, \mathbf{n_3}]^{\mathbf{T}}$$

$$\mathbf{p} \cdot \mathbf{n} = 0$$



Lie Groups and AX=XB

If $A^i = XB^iX^{-1}$ and we define $\mathbf{l}_{A^i}(t) = \mathbf{p}_{A^i} + t\mathbf{n}_{A^i}$



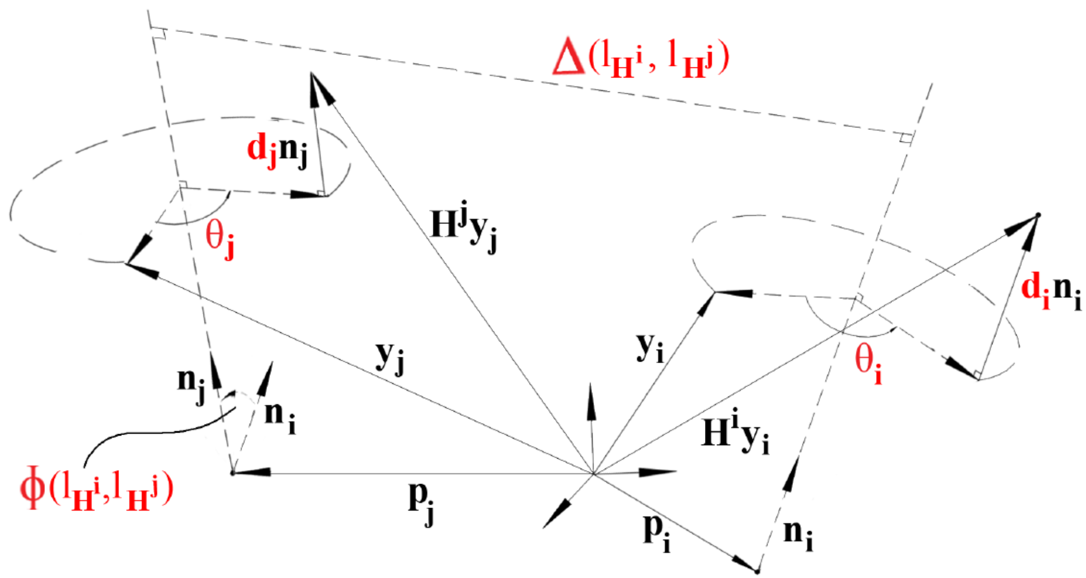
Where:

$$\Delta(\mathbf{l}_{A^{i_1}}, \mathbf{l}_{A^{i_2}}) = \frac{|[\mathbf{n}_{A^{i_1}}, \mathbf{n}_{A^{i_2}}, \mathbf{p}_{A^{i_2}} - \mathbf{p}_{A^{i_1}}]|}{\|\mathbf{n}_{A^{i_1}} \times \mathbf{n}_{A^{i_2}}\|}$$

$$\cos\phi(\mathbf{l}_{A^{i_1}}, \mathbf{l}_{A^{i_2}}) = \mathbf{n}_{A^{i_1}} \cdot \mathbf{n}_{A^{i_2}}$$

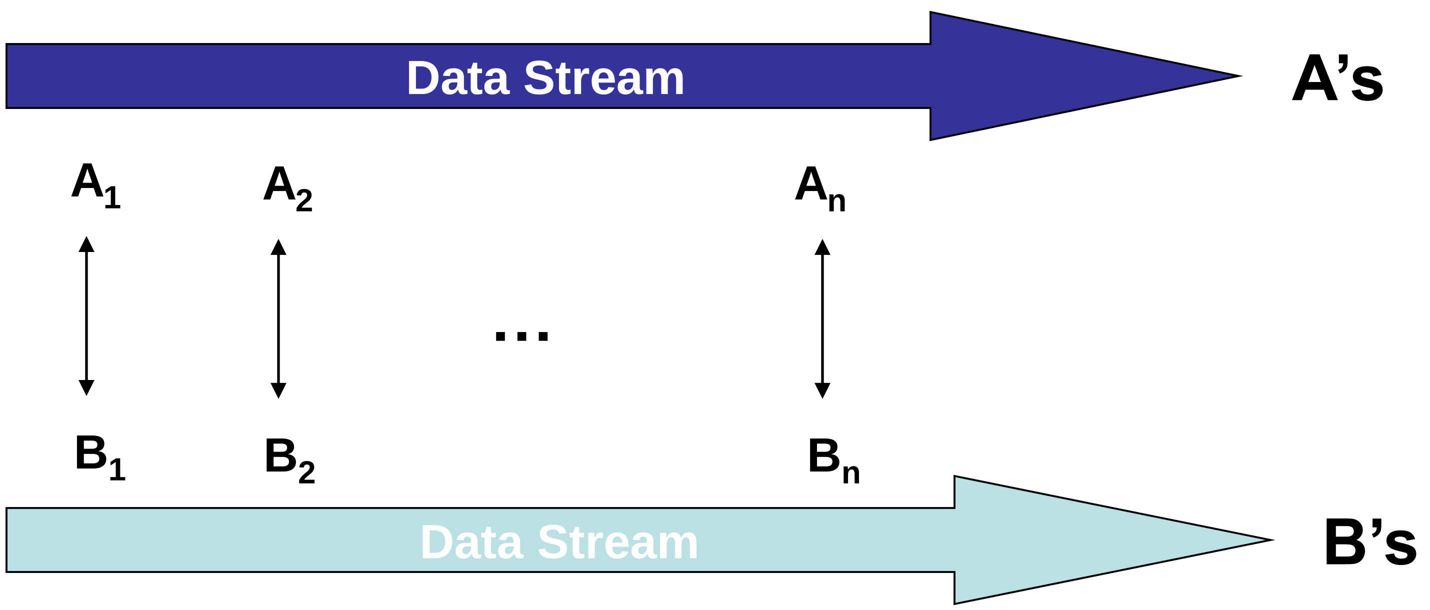
$$\sin\phi(\mathbf{l}_{A^{i_1}}, \mathbf{l}_{A^{i_2}}) = \Delta(\mathbf{l}_{A^{i_1}}, \mathbf{l}_{A^{i_2}})^{-1}[\mathbf{n}_{A^{i_1}}, \mathbf{n}_{A^{i_2}}, \mathbf{p}_{A^{i_2}} - \mathbf{p}_{A^{i_1}}]$$

AX=XB and the Euclidean Invariants



1) $\theta_{A^{i_1}} = \theta_{B^{i_1}}$ and $\theta_{A^{i_2}} = \theta_{B^{i_2}}$
2) $d_{A^{i_1}} = d_{B^{i_1}}$ and $d_{A^{i_2}} = d_{B^{i_2}}$
3) $\phi(\mathbf{l}_{A^{i_1}}, \mathbf{l}_{A^{i_2}}) = \phi(\mathbf{l}_{B^{i_1}}, \mathbf{l}_{B^{i_2}})$
4) $\Delta(\mathbf{l}_{A^{i_1}}, \mathbf{l}_{A^{i_2}}) = \Delta(\mathbf{l}_{B^{i_1}}, \mathbf{l}_{B^{i_2}})$

Sometimes Data Streams are Asynchronous



Probability Theory on SE(3)

The mean and covariance of a probability density function, $f(H)$, can be defined as

$$\int_{SE(3)} \log(M^{-1}H) f(H) dH = \mathbb{O}$$

and

$$\Sigma = \int_{SE(3)} \log^{\vee}(M^{-1}H)[\log^{\vee}(M^{-1}H)]^T f(H) dH$$

For comparisons of different concepts of mean and covariance on SE(3), see Chirikjian, G., "Stochastic Models, Information Theory, and Lie Groups" Birkhauser, 2011

Our "Batch" Method

$$A_i X = X B_i \quad \Rightarrow \quad (\delta_{A_i} * \delta_X)(H) = (\delta_X * \delta_{B_i})(H)$$

Because real-valued functions can be added and convolution is a linear operation on functions, all n instances can be written in to a single equation of the form

$$(f_A * \delta_X)(H) = (\delta_X * f_B)(H) \ \text{ where}$$

$$f_A(H) = \frac{1}{n}\sum_{i=1}^{n} \delta(A_i^{-1}H) \text{ and } f_B(H) = \frac{1}{n}\sum_{i=1}^{n} \delta(B_i^{-1}H)$$

We can normalize the functions to be probability density functions (pdfs):

$$\int_{SE(3)} f_A(H)dH = \int_{SE(3)} f_B(H)dH = 1$$

Our “Batch” Method

Since the mean of $\delta_X(H)$ is $M_X = X$ and its covariance is the zero matrix we can write our “Batch” method formulation

$$(\delta_{A_i} * \delta_X)(H) = (\delta_X * \delta_{B_i})(H)$$

$$f_A(H) = \frac{1}{n}\sum_{i=1}^{n}\delta(A_i^{-1}H) \text{ and } f_B(H) = \frac{1}{n}\sum_{i=1}^{n}\delta(B_i^{-1}H)$$

$$M_{1*2} = M_1 M_2 \text{ and } \Sigma_{1*2} = Ad(M_2^{-1})\Sigma_1 Ad^T(M_2^{-1}) + \Sigma_2$$

Batch Method “AX=XB” Equations:

(1) $\boxed{M_A X = X M_B}$ (2) $\boxed{Ad(X^{-1})\Sigma_A Ad^T(X^{-1}) = \Sigma_B}$

Topic 4:

Modularity in Manufacturing and Autonomous Field Robots

A Modular Manufacturing System

An architecture for universal construction via modular robotic components

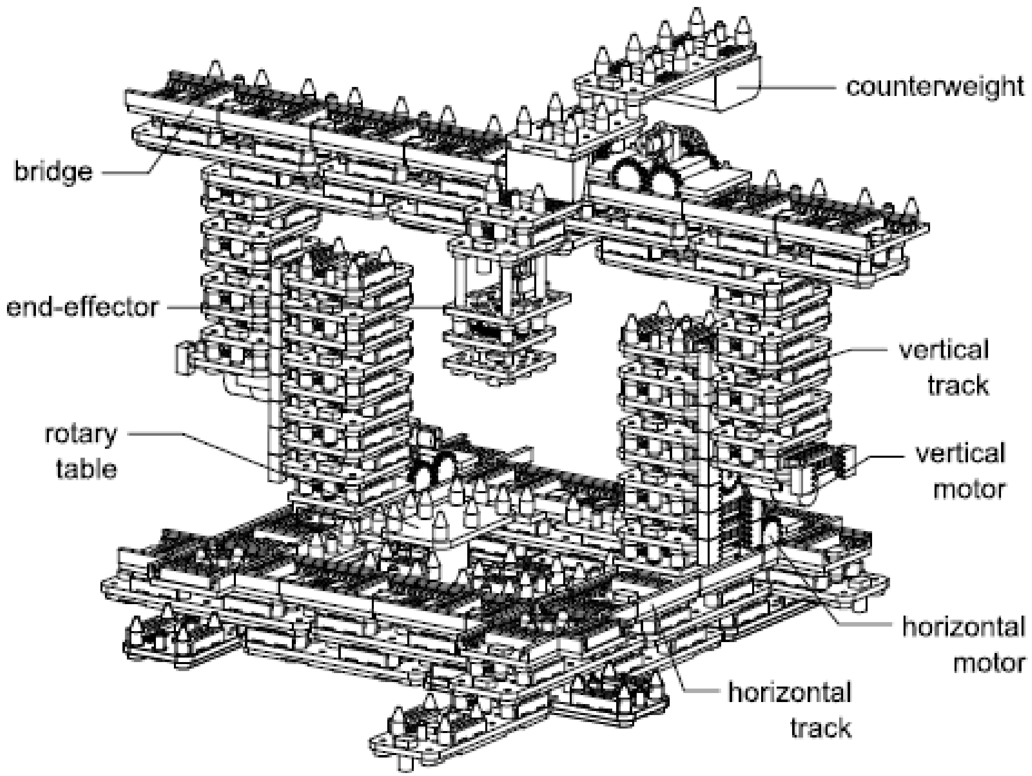


Fig. 1. The constructor is a general purpose 3-axis manipulator, with access to a rotary table for part re-orientation. The constructor workspace allows it to assemble indefinite extensions of track.

Robotics and Autonomous Systems 62 (2014) 945–965

architecture for universal construction via modular robotic An components

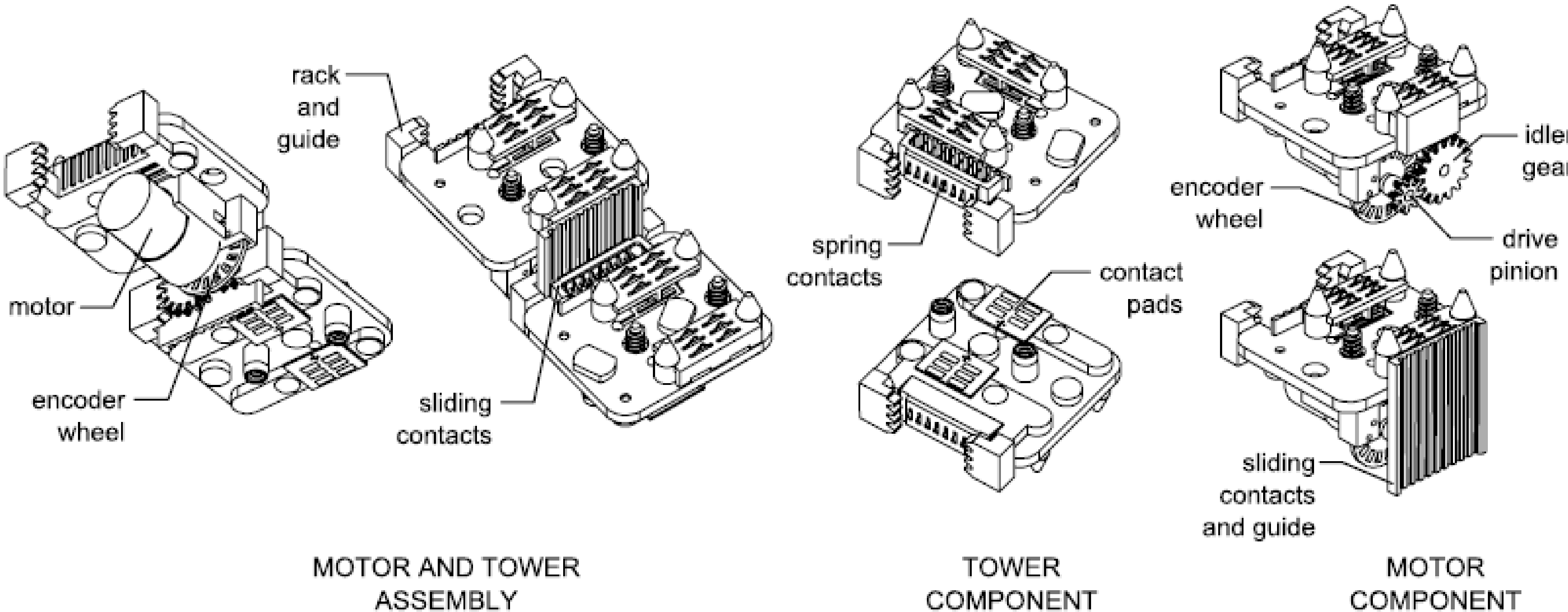


Fig. 18. Key features of the vertical motor and track components. These are Part Types 15 and 12, respectively, as seen in Fig. 2.

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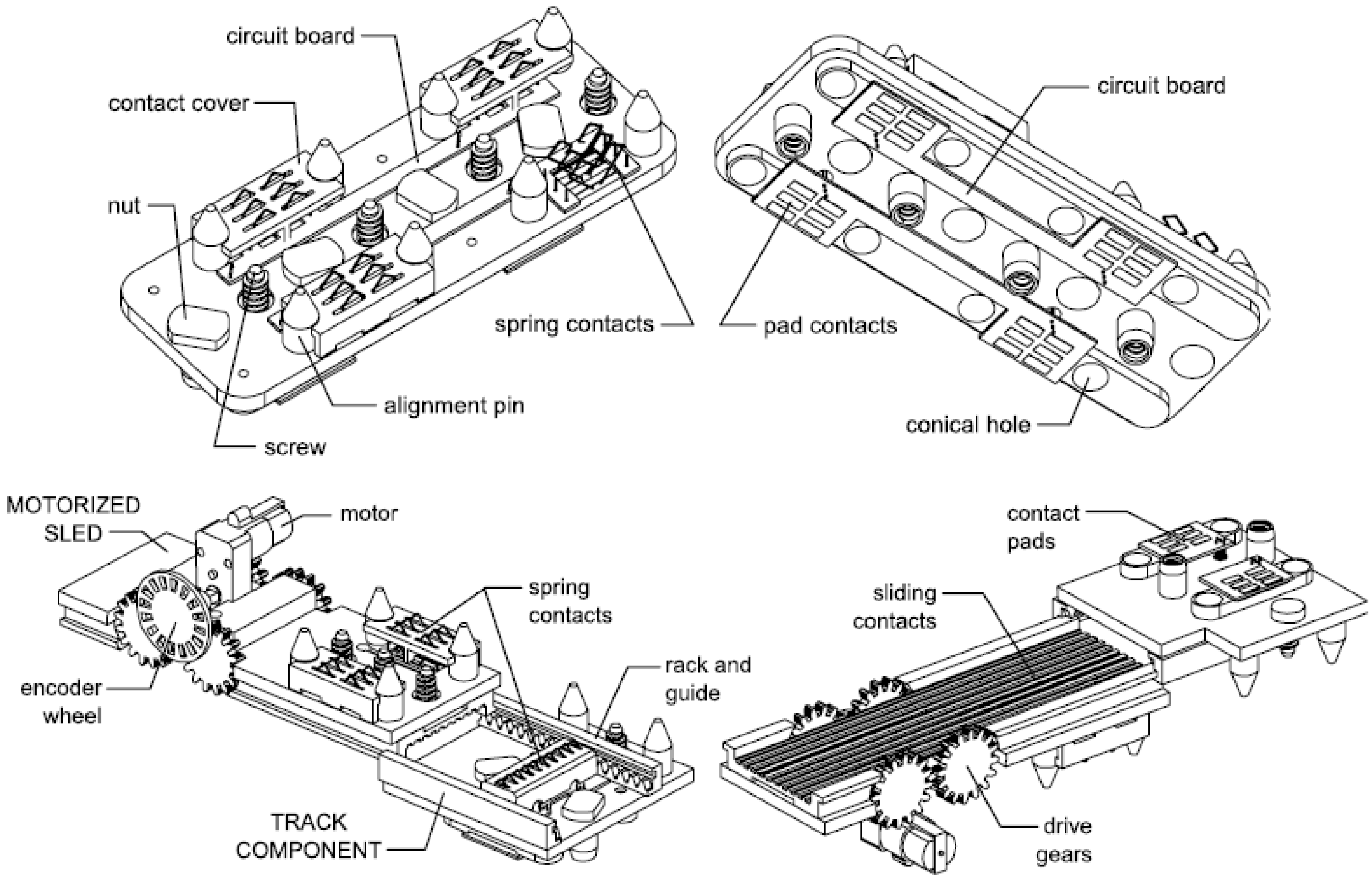
An architecture for universal construction via modular robotic components



Robotics and Autonomous Systems 62 (2014) 945–965

An architecture for universal construction via modular robotic components

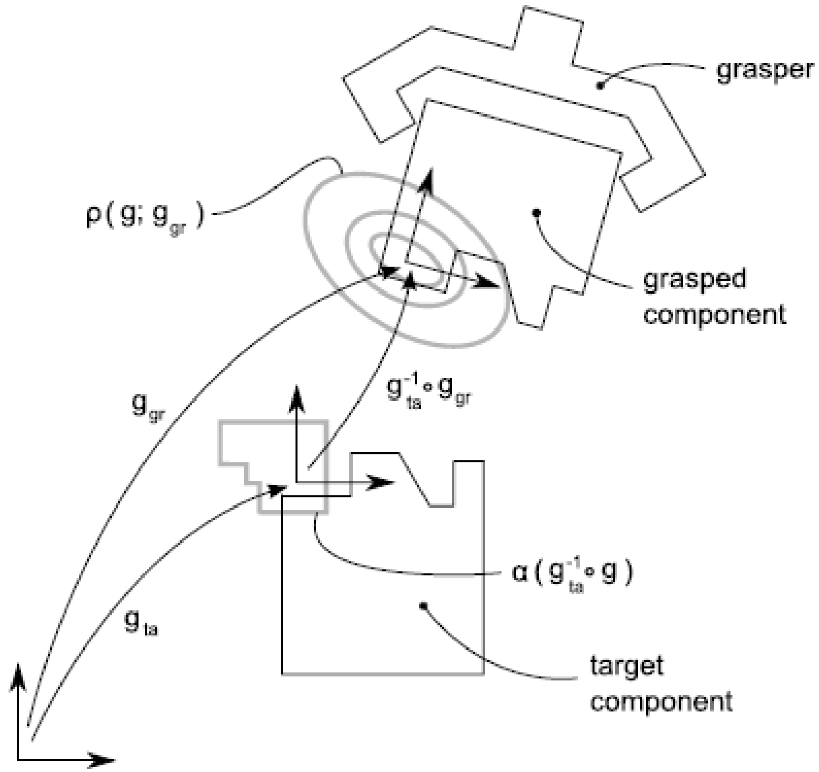


Fig. 8. Uncertainty in positioning of the grasper is represented by a probability density function $\rho(g; g_{gr})$. The function $\alpha(g_{ta}^{-1} \circ g)$ is the probability of a successful connection given a relative displacement between grasper and target. The independent variable is g.

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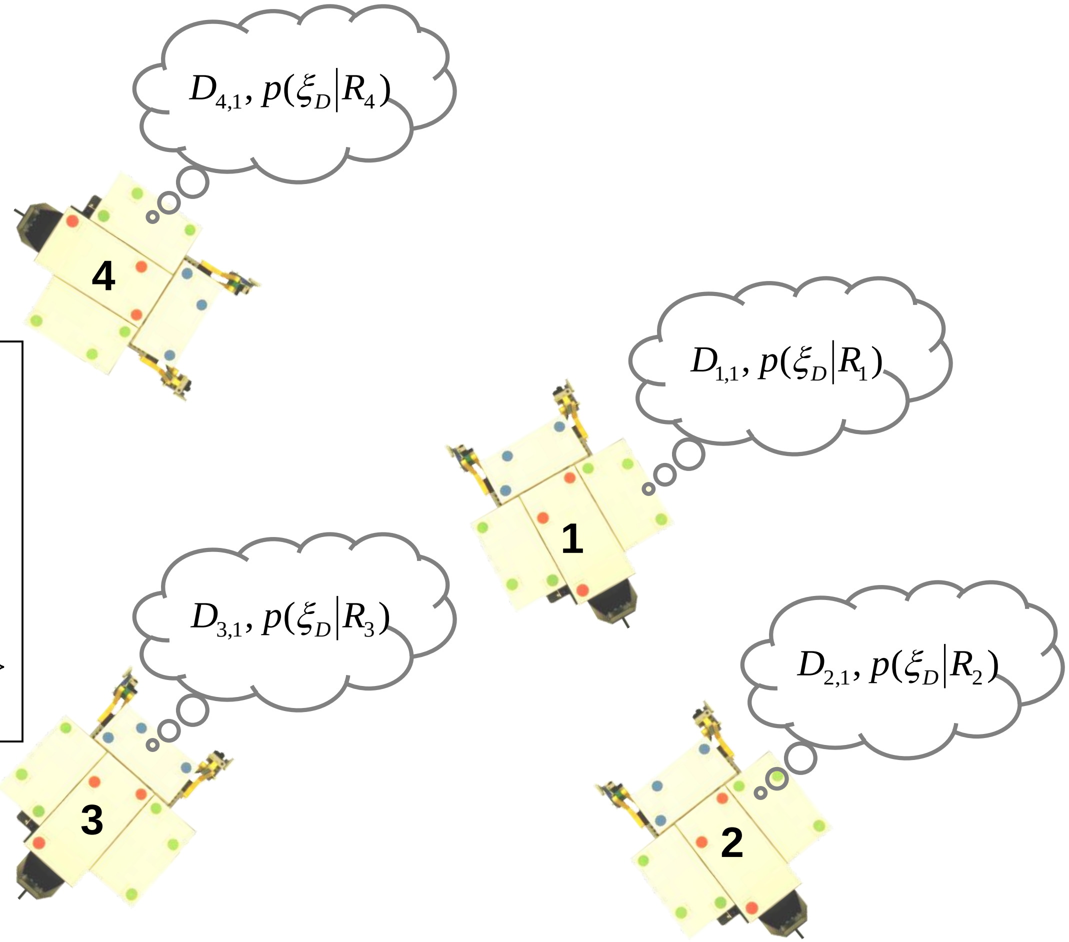
Endowing Teams of Mobile Robots with Greater Robustness

Group Diagnosis

$$p(\xi_D | R_i) << 1 \quad \forall i \neq 1$$

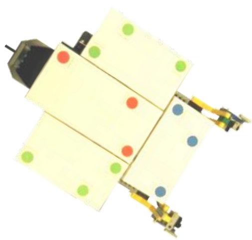
$$p(\xi_D | R_1) < 1$$

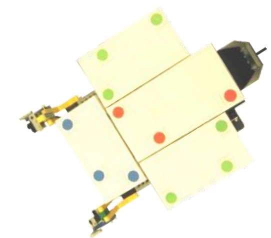
$$\boxed{\begin{aligned} & p(\xi_D | R_1 \,\&\, R_2 \,\&\, \ldots) \\ & \quad = \prod_{i=1}^{4} p(\xi_D | R_i) \\ & \quad < p(\xi_D | R_j) \quad \forall j \in \{1,2,\ldots\} \end{aligned}}$$

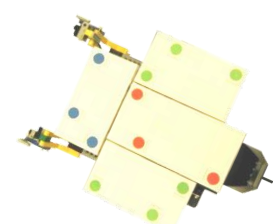


Testbed Diagnosis Routine

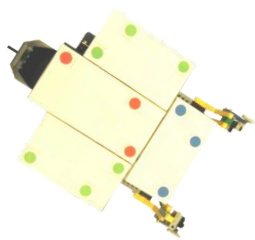
$$y_1 = \begin{pmatrix} e_1 \\ e_2 \\ \theta \end{pmatrix}$$



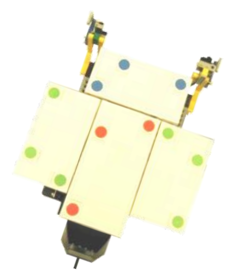


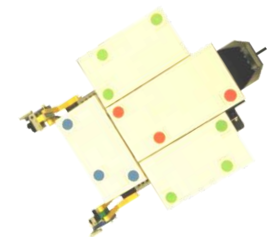


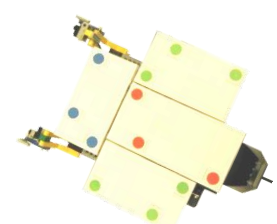
Testbed Diagnosis Routine



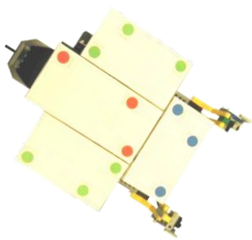
$$y_2 = \begin{pmatrix} e_1 \\ e_2 \\ \theta \end{pmatrix}$$



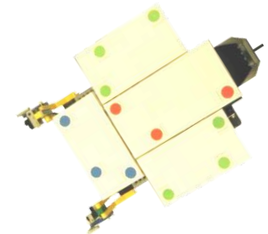




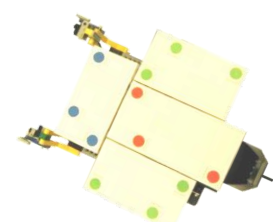
Testbed Diagnosis Routine



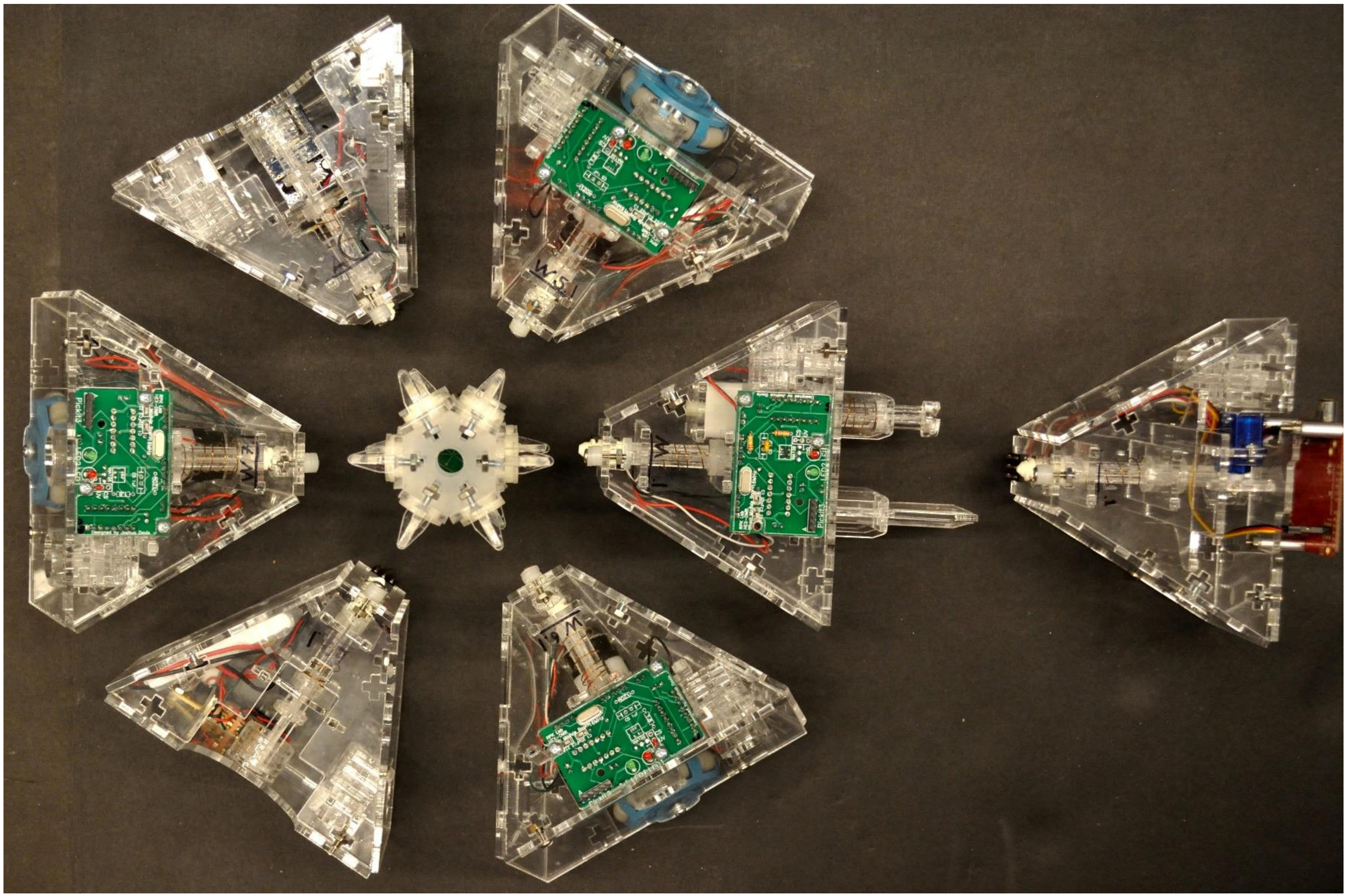
$$y_6 = \begin{pmatrix} e_1 \\ e_2 \\ \theta \end{pmatrix}$$



$$Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_6 \end{pmatrix}$$



Hex DMR II



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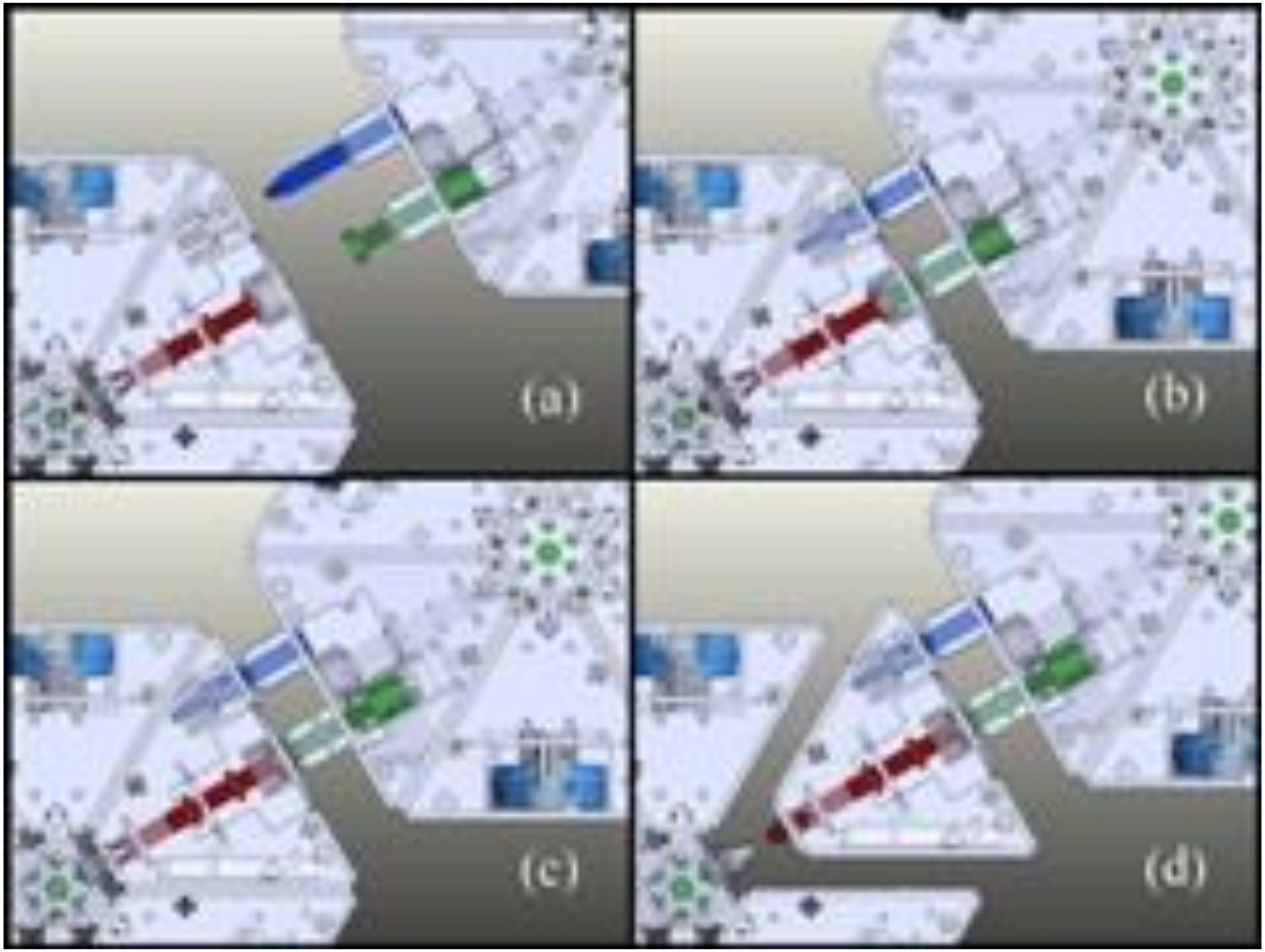
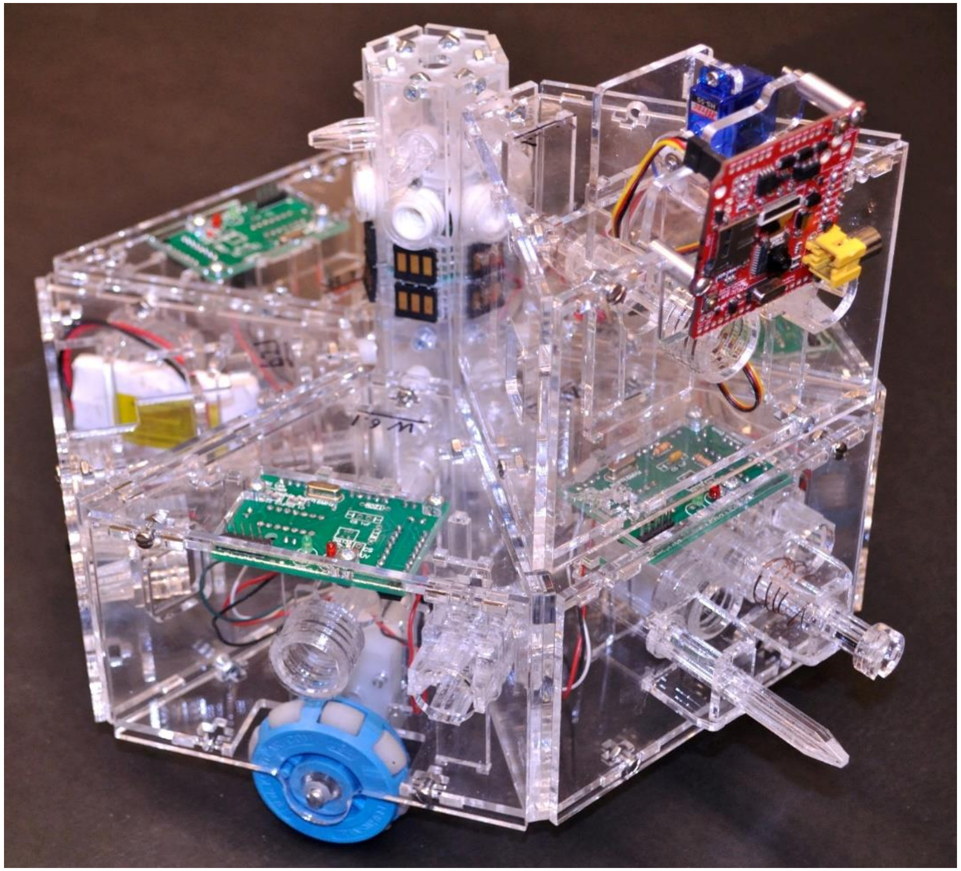
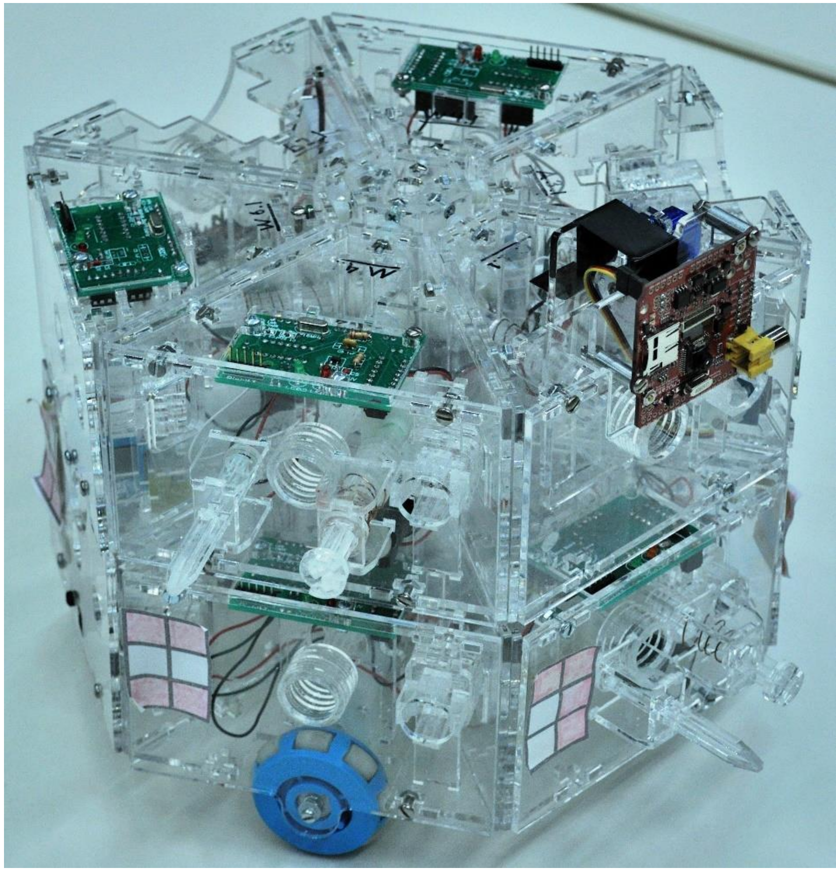


The Hex DMR II





References

1) Kutzer, M.D.M., Armand, M., Lin, E., Scheidt, D., Chirikjian, G.S., ``Toward Cooperative Team-Diagnosis in Multi-robot Systems,''
International Journal of Robotics Research 27: 1069-1090, Sept. 2008.

2) Chirikjian, G.S., ``Information-Theoretic Inequalities on Unimodular Lie Groups,'' Journal of Geometric Mechanics, June 2010.

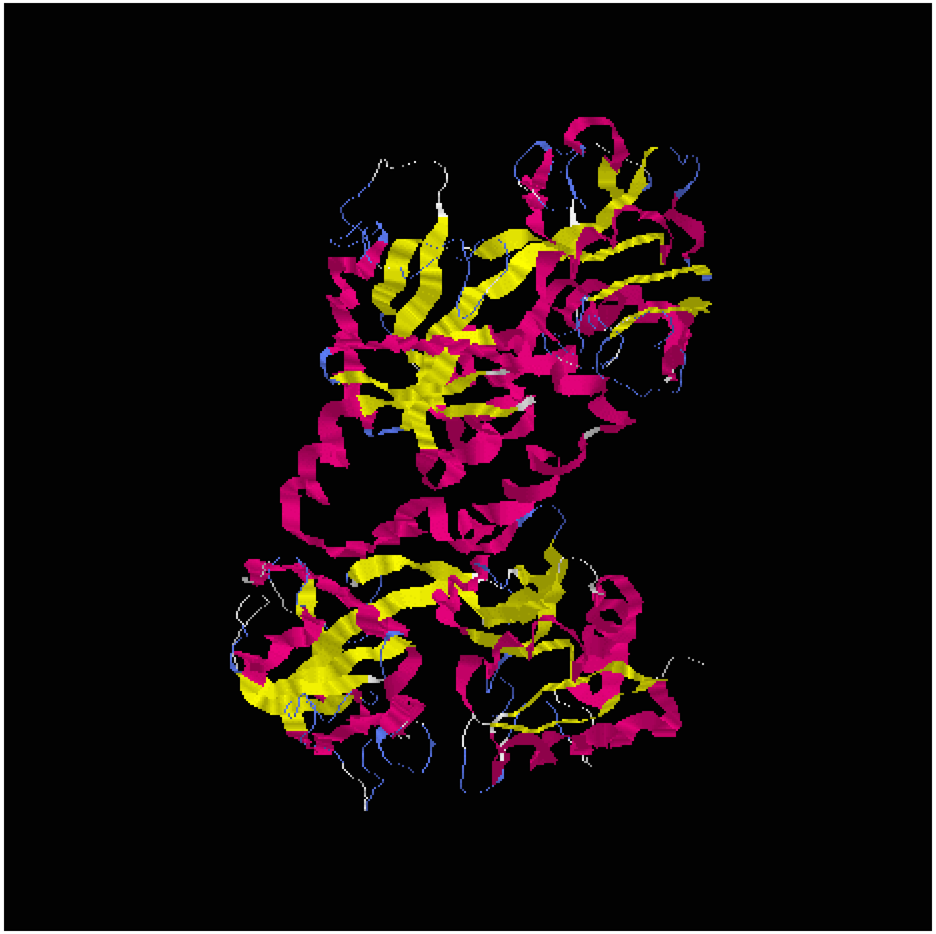
3) Park, W., Liu, Y., Moses, M., Chirikjian, G.S., ``Kinematic State Estimation and Motion Planning for Stochastic Nonholonomic Systems Using the Exponential Map,'' Robotica, 26(4), 419-434. July-August 2008

4) Lee, K., Moses, M., Chirikjian, G.S.,. ``Robotic Self-Replication in Partially Structured Environments: Physical Demonstrations and Complexity Measures,'' International Journal of Robotics Research, 27(3-4): 387-401, March 2008.

Topic 5:

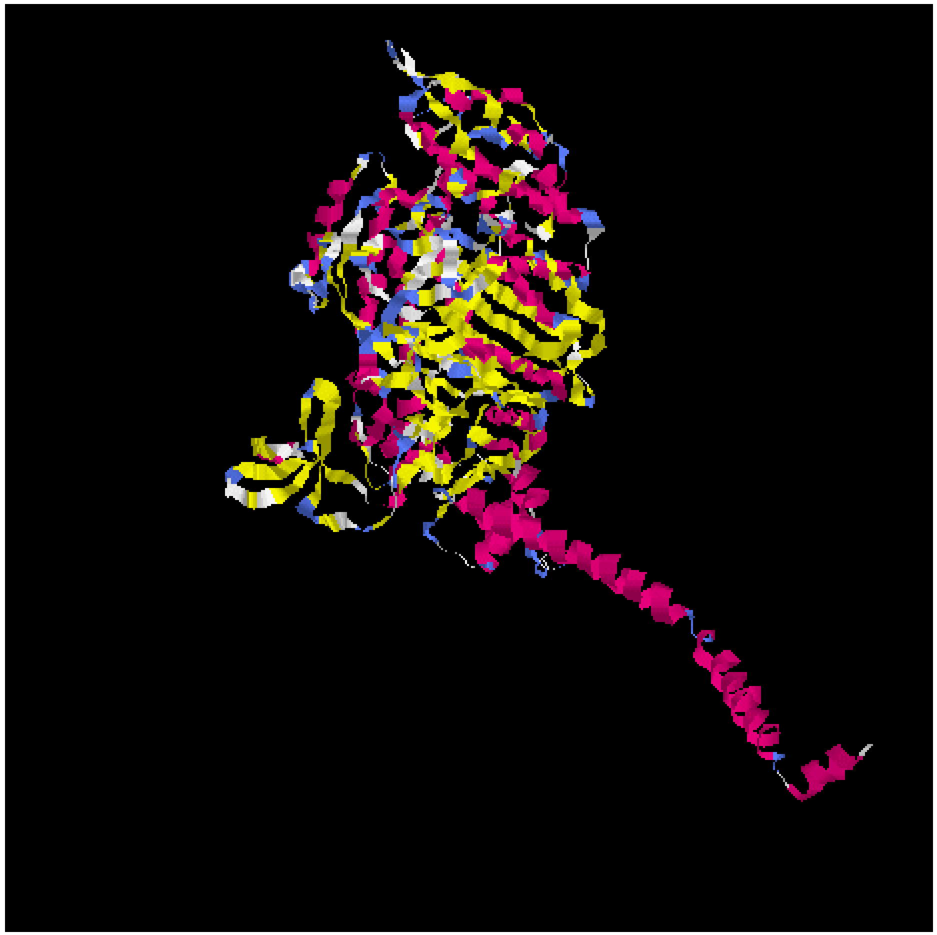
Proteins as Machines

Lactoferrin Transition from 1lfg.pdb to 1lfh.pdb



Calcium ATPase Transition (1KJU.pdb to 1EUL.pdb)

Scallop Myosin Transition from 1DFL.pdb to 1DFK.pdb



Summary

Our lab does a lot of different things, including:

Mechanical/Robot Design (snakes, spherical motors, modular self-reconfigurable robots, self-replicating robots, etc.);

Applied Mathematics;

Information-Driven Motion in Robotics;

Medical Image Registration;

Acknowledgements

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Robotic Self-Replication and Self-Repair (see videos)