Kinematics of a Metamorphic Robotic System

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Abstract

A *metamorphic* robotic system is a collection of mechatronic modules, each of which has the ability to connect, disconnect, and climb over adjacent modules. A change in the macroscopic morphology results from the locomotion of each module over its neighbors. That is, a metamorphic system can dynamically self-reconfigure. Metamorphic systems can therefore be viewed as a large swarm of physically connected robotic modules which collectively act as a single entity. What separates metamorphic systems from other reconfigurable robots is that they possess all of the following properties: (1) self-reconfigurability without outside help (2) a large number of homogeneous modules (3) physical constraints ensure contact between modules. In this paper, the kinematic constraints governing a particular metamorphic robot are addressed.

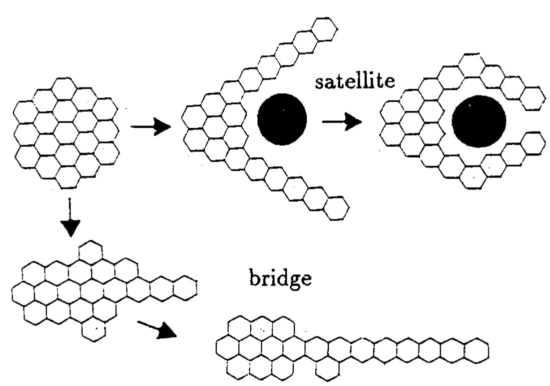

Figure 1: Applications of Metamorphic Robots

1 Introduction

A *metamorphic* robotic system is a collection of independently controlled mechatronic modules, each of which has the ability to connect, disconnect, and climb over adjacent modules. Each module allows power and information to flow through itself and to its neighbors. A change in the metamorphic manipulator morphology (i.e., a change in the relative location of modules within the collection) results from the locomotion of each module over its neighbors. Thus a metamorphic system has the ability to dynamically self-reconfigure. Changes in configuration within a given morphology are achieved by changing joint angles, as is the case for standard (fixed-morphology) robotic manipulators.

Metamorphic systems can therefore be viewed as a large swarm (or colony) of connected robots which collectively act as a single entity. What separates metamorphic systems from other reconfigurable robots is that they possess all of the following properties: (1) self-reconfigurability without outside help; (2) a large number of modules (in the limiting case the configuration can be thought to approximate a continuous 'blob'); (3) physical constraints ensure contact between modules; (4) uniformity and completeness of module function.

Potential applications of metamorphic systems include : (1) obstacle avoidance in highly constrained and unstructured environments; (2) 'growing' structures composed of modules to form bridges, buttresses, and other civil structures in times of emergency; (3) envelopment of objects, such as recovering satellites from space. Some of these applications are shown in Figure 1. At the level of micro-machines, one could imagine such robots being used to provide structural reinforce-

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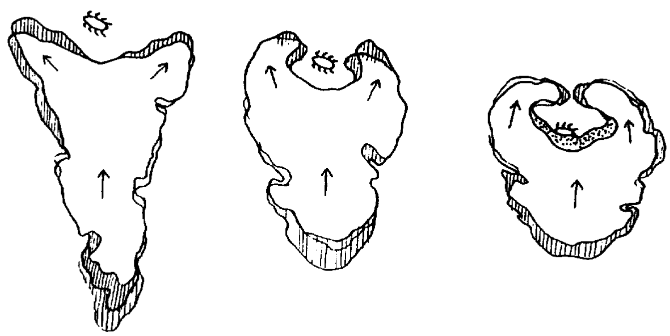

Figure 2: An Amoeba (Depicted in [Sch20])

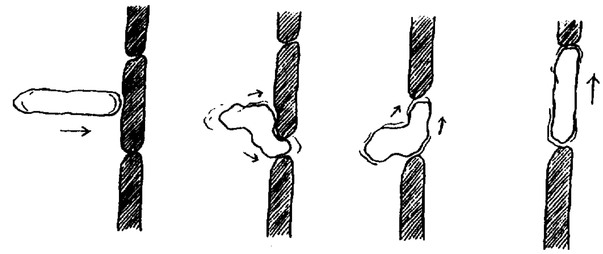

Figure 3: A Slime Mold (Depicted in [Bo67])

ment in the organs of the human body, or to surround and isolate tumors.

This paper addresses issues in the kinematics of a particular metamorphic design. In Section 2, a brief review of the related literature is presented. In Section 3, the kinematics of a planar three degree-of-freedom module is explained. These modules are each kinematically sufficient, which allows them the freedom to 'walk' over each other. The kinematics of locomotion associated with this design is also examined. Section 4 develops a coordinate system to describe the lattice formed by multiple modules, and ennumerates its properties. This section also develops local heuristic rules which ensure that the system remains connected.

2 Literature Review

The idea of a metamorphic robotic systems differs from related concepts presented in the literature. Three types of modular reconfigurable robotics systems have been proposed in the literature: (1) robots in which modules are reconfigured using external intervention [BeZL89, CoLDB92, KeK88, Sci85, W86]; (2) cellular robotic systems in which a heterogeneous collection of independent specialized modules are coordinated [Be88, BeW91, FuN88, FuK90, FuKH91, HaW88]; (3) swarm intelligence in which there are generally no physical connections between modules [HaB92, HaB91, AsOJMIE91]. The concept of a metamorphic system differs from all of the above because modules are homogeneous in form and function, contact between modules must always occur, and self-reconfiguration is possible.

As the number of modules in a metamorphic system approaches infinity, the manipulator could be viewed as a 'mechatronic amoeba' [Je73, Sch20, Bo67] (see Figure 2 for a schematic of a real amoeba) because the manipulator takes on a continuous appearance. Figure 3 shows how a slime mold can reconfigure itself. Thus, the idea of metamorphic structures is not foreign to the natural world.

The next section discusses issues in the mechanical design and kinematics of a particular metamorphic module.

3 Module Kinematics

This section addresses issues in the mechanical design and kinematics of planar mechatronic modules used as components of a metamorphic robot. The most intrinsic, or core, properties to the idea of metamorphic robotic system are:

1. All modules should have the same physical structure, so that uniform treatment of modules in the planning problem is possible.
2. Symmetries in the mechanical structure of the modules must be such that they can be easily 'close-packed' ,i.e., fill planar and spatial regions without gaps. In this way, a lattice of modules is formed for any task.
3. The modules must each be kinematically sufficient with respect to the task of locomotion, i.e., they must have enough degrees of freedom to be able to 'walk' over adjacent modules.

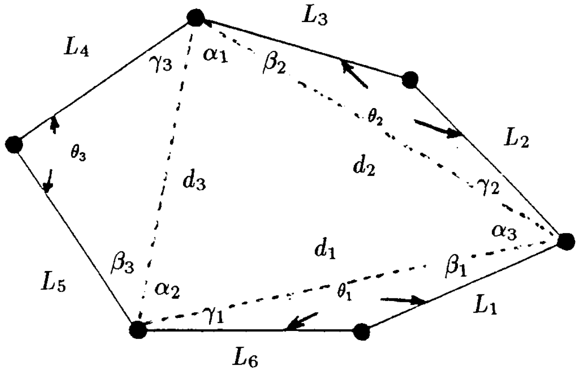

Figure 4: An Individual Module

4. A means by which modules are made to adhere to adjacent modules must be divized. In this way the collection of modules becomes a single physical object.

While an infinite variety of module designs satisfy the above conditions, one particular class is discussed here. These are planar closed-loop mechanisms.

In order to satisfy condition (1) above, regular polygonal module designs were chosen, i.e., closed loop mechanisms with uniform link lengths. In this way, the modules are not only uniform, but also possess a multiplicity of rotational symmetries. Condition (2) then reduced to finding what regular polygons close-pack the plane. This became a choice between the triangle, square, and hexagon, i.e., three, four, or six bar linkages. Since the triangle has zero mobility, condition (3) could only be satisfied with a square or hex. The hex was finally chosen (see Figure 4), because its three degrees of mobility are superior for locomotion. Condition (4) is satisfied by specifying that alternating links in the hex have opposite 'polarity,' i.e., male/female connectors, magnetic fields of opposite signs, etc. It is assumed that locomotion of the module is implemented by a combined rigid body rotation and shape transformation produced by changing module joint angles. The resulting 'rolling' type of locomotion is shown in Figure 5 along with the alternating polarity links. In this way, as a given hexagonal module locomotes over a collection of other modules, opposite signs will always be in contact. Polarity matching is ensured since each module has an even number of links and the boundary of any collection of modules will also have alternating polarities.

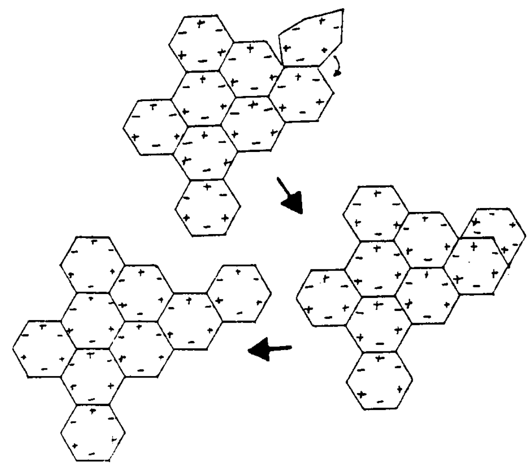

Figure 5: The Reconfiguration Process

Because the six bar linkage design has three degrees of mobility, three motors are required to specify the module geometry completely. This is accomplished by placing motors at alternating vertices. This also makes for a design with nice symmetries.

In the general case, the kinematics of a six-bar linkage can be derived using trigonometric arguments with the constructions shown in Figure 4. Without explicitly stating the cumbersome equations, it suffices to say that for given link lengths, L_i and joint angles θ_i, the Law of Cosines will supply the lengths d_i, and therefore α_i as well. Then the Law of Sines will supply γ_i and β_i. Then all the interior angles of the linkage are completely specified for the given the link lengths, L_i, and actuated joint angles, θ_i.

The fact that the links of each module have finite thickness must also be taken into account. Two ways to do this are: (1) as a module moves over a 'terrain' composed of other modules, the terrain can flex so as to ensure matching of module connectors; (2) the module links can be designed to be extensible, i.e., they can expand or contract relative to their nominal length, so that they can conform to rigid hexes composing the terrain.

For the sake of illustrating how joint angles are chosen as a function of time, the idealized case in which the link thicknesses are zero will be considered here. In this way, the locomotion process can be exemplified without examining the routine yet messy mathematics resulting from the complications of extensible links or flexing neighbor modules. While these problems must be addressed for real implementation, they do not add to the fundamental understanding of lattice kinematics discussed in the following section in which each hex is given integer coordinates (i, j).

Therefore, consider the idealized problem: 'How is the rolling motion of a module from coordinates (i, j) to any of its neighbors to be described ?' Given that there is a heuristic rule, or cost function, which will determine which move is desirable, we still need to encode this basic kinematic information. That is, a motion planner will provide discrete information such as 'move from (i, j) to $(i+1, j+1)$.' The module kinematics problem then consists of two parts: (1) Determine if the module is to roll clockwise or counterclockwise to get to the new location; (2) Determine a useful time evolution of joint trajectories.

In order to solve these problems, the concept of a 'leading vertex' is used. Basically, the leading vertex is the vertex of the hex which is to move which is also shared by the goal hex and the adjacent module. For reasons discussed in the following section, there will only be one module which is adjacent to both a locomoting module and a free space (See the discussion of contiguous neighbors). Therefore, the leading vertex, which would be the center of rotation if the locomoting hex were rigidly rotated into the new space, is uniquely determined. The interior angle of the leading vertex is denoted by θ_1. The angles θ_2 and θ_3 are defined as the remaining alternating vertices. This numbering depends on whether the module must rotate clockwise or counterclockwise into its new space, Note that the angles θ_i will *not* correspond to particular motors because joints will constantly be renumbered as the leading vertex changes. However, these angles are used to determine what the joint angles at the motors must be.

There are an infinite number of acceptable joint trajectories $\theta_i(t)$ which will solve the problem. Likewise, there are a large number of optimality criteria which could be imposed to determine appropriate joint trajectories. Due to space limitations, only those constraints which are absolutely necessary are included here with a peripheral discussion of trajectories. Namely, at an initial time t_1 before the locomotion procedure has started, and at a time t_2, after the locomotion is finished, it must be true that $\theta_i(t_1) = \theta_i(t_2) = 2\pi/3$. In the middle of the locomotion procedure, $\theta_1((t_1+t_2)/2) = 4\pi/3$ in order for the module to connect to the new space. The particular trajectory which links the initial, middle, and end values of θ_1 are somewhat arbitrary, e.g. splines. The angles θ_2 and θ_3 are typically defined to vary 'as little as possible' while the leading vertex undergoes large changes in angle.

4 Lattice Kinematics

This section addresses issues in the macroscopic kinematics of metamorphic robotic systems. In Subsection 4.1 a coordinate system for describing macroscopic configuration is presented and its metric properties are ennumerated. In Subsection 4.2, constraints on the connectivity of modules are derived within this coordinate system. Finally, in Subsection 4.3 the kinematic constraints on the lattice are demonstrated in the context of an envelopment task.

4.1 Defining Distance

The plane can be decomposed into hexagonal units. These units are either filled by modules or obstacles or remain empty. The whole plane is then viewed as a lattice of hexagonal spaces which are either empty or filled. In order to completely describe the position of each module within this lattice, it is desirable to coordinatize the lattice. One way to do this is to use the *Miller-Bravais* framework used in materials science and crystallography. In this framework, a redundant set of coordinates (in this case, 3) are used to describe the position of each point. An alternate framework is used here. This is depicted in Figure 6. By denoting the origin as the point $(0, 0)$, and using the numbering convention illustrated in this figure, every point in the plane is given a unique set of coordinates, as shown in Figure 7.

In order for the kinematics of the lattice to be complete, the most important geometric quantity of all must be defined: distance. A proper distance (or metric) function between points A and B will satisfy the following properties:

$$d(A, B) > 0 \quad if A \neq B \quad d(A, A) = 0$$

$$d(A, B) = d(B, A)$$

$$d(A, B) + d(B, C) \geq d(A, C).$$

The two most well known metrics in $\mathbf{R}^2$ are the Euclidean metric, which defines the length of the straight line segment connecting two points in the plane, and the 'Taxicab' (also called the Manhattan) metric in

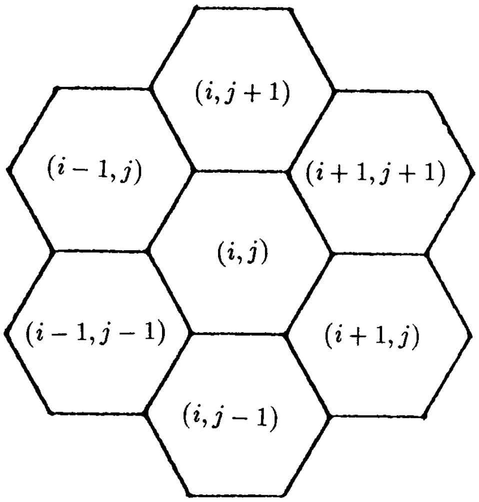

Figure 6: Numbering Convention

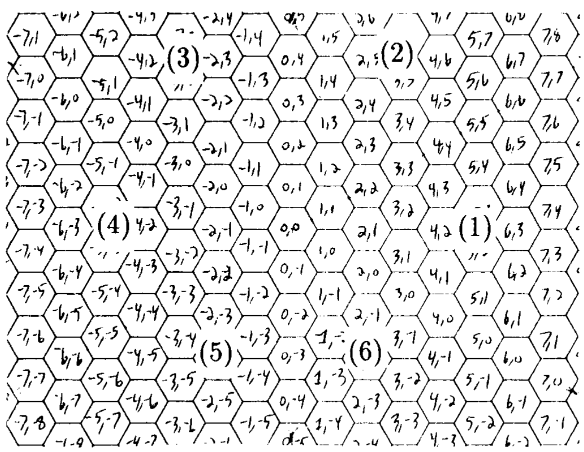

Figure 7: The Resulting Coordinate Lattice

which case the absolute values of the difference between the x coordinates are added to that of the y coordinates. The latter of these gets its name from the distance which one must travel in an ideal city in which every block is the same size and every street is perpendicular to every other.

Within the coordinate system which is generated from the patch shown in Figure 6, and demonstrated in Figure 7, the Euclidean metric is

$$\delta_E(\Delta i, \Delta j) = \left((\Delta i)^2 + (\Delta j)^2 - \Delta i \Delta j\right)^{\frac{1}{2}},$$

where $A = (i_1, j_1)$, $B = (i_2, j_2)$, $\Delta i = i_1 - i_2$, and $\Delta j = j_1 - j_2$.

The extra term added to the usual $\left((\Delta i)^2 + (\Delta j)^2\right)^{\frac{1}{2}}$ results from the fact that one of the coordinate axes is skewed so that there is not a ninety degree angle between the axes. The above formula for δ_E is calculated easily from trigonometry. These axes are indicated by the solid lines in Figure 7. Because of this fact, the taxicab norm is exceedingly complicated in this coordinate system. However, we would like to have an analogous system which will measure the true distance a module must travel within the regular lattice formed by all other modules if it is to roll from one lattice space to another.

This is achieved quite simply by taking the difference between the coordinates of the two points and treating the difference as a vector centered at the origin of Figure 7. Depending on which sextant this vector falls in, a different distance function is used as stated below:

$$\delta(\Delta i, \Delta j) = |\Delta i| \quad \text{in} \quad 1, 4$$

$$\delta(\Delta i, \Delta j) = |\Delta j| \quad \text{in} \quad 2, 5.$$

$$\delta(\Delta i, \Delta j) = |\Delta i| + |\Delta j| \quad \text{in} \quad 3, 6$$

It is conceptually trivial to show that these satisfy the definition of a metric given previously by direct calculation for each case. Furthermore, it is easy to see that this metric is bounded below by the Euclidean metric as demonstrated for Sextant 1 (in which $-\Delta i \leq \Delta j - \Delta i \leq 0$) below:

$$\delta_E^2 = |\Delta i|^2 + |\Delta j|^2 - \Delta i \Delta j$$

$$= |\Delta i|^2 + \Delta j(\Delta j - \Delta i) \leq |\Delta i|^2 = \delta^2.$$

Furthermore, if we compare the minimal value of the Euclidean metric with the new one presented above for fixed Δi, we see that

$$\frac{\partial \delta_E^2}{\partial(\Delta j)} = 2\Delta j - \Delta i = 0,$$

indicating that

$$\min(\delta_E^2) = \frac{3}{4}(\Delta i)^2 = \frac{3}{4}(\delta)^2, \quad \text{or} \quad \delta_E^2 \geq \frac{3}{4}\delta^2.$$

Thus, it is clear that for $(\Delta i, \Delta j)$ in sextant 1, the new metric is bounded below and above as follows:

$$\delta_E \leq \delta \leq \frac{2}{\sqrt{3}}\delta_E.$$

Furthermore, it can be reasoned from symmetry that this relationship holds always.

(a)

(b)

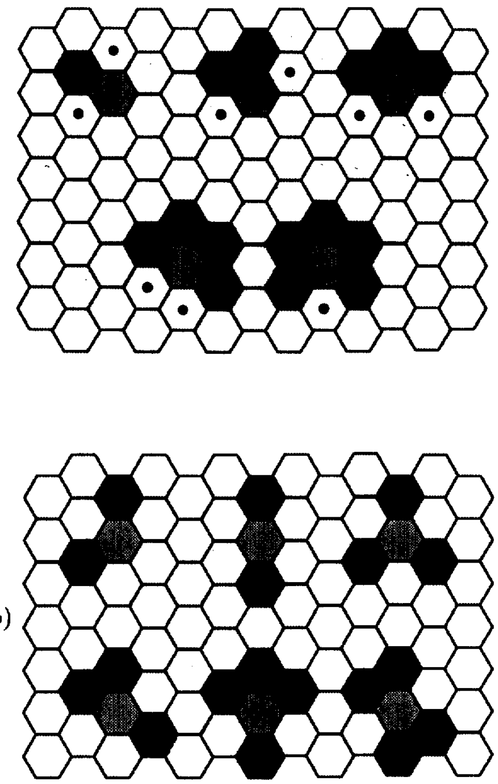

Figure 8: Contiguous and Noncontiguous Neighbors

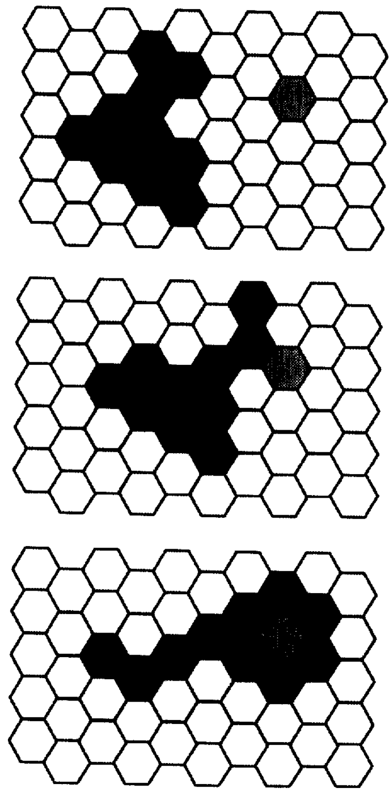

Figure 9: Convergence to Object

4.2 Motion Constraints

An example of a set of consistent (and intuitive) kinematic constraints are:

- Modules can only move into spaces which are not already occupied by hubs or other modules.
- Every module must remain connected to at least one other module or hub, and at least one of the modules must stay connected to the hub from which the collection of modules originated.

These rules can be enforced in any number of ways including developing an evolving graph to capture the changing topology of a metamorphic system. While such an approach is attractive, it has the drawback of requiring global information. Instead, the above constraints will be enforced by using a simple local rule: *A module is only allowed to move if it does not have, or will not gain, noncontiguous neighbors.* Figures 8(a) and 8(b) respectively illustrate situations in which the neighbors of the center module are contiguous and noncontiguous. In fact, up to rotation and reflection, these are the only possible cases. Clearly, when neighbors are contiguous to each other, the motion of a module via 'rolling' will not isolate any module.

4.3 Applications to Motion Planning

In this section, the motion planning problem is addressed for the particular problem of using a metamorphic robotic system to envelop an object. This is achieved by simply applying the hierarchy of kinematic rules which govern the motion of modules within the lattice, in combination with an artificial potential field. Each module feels an artificial force to move which is inversely proportional to its distance from the goal within the δ metric. When the closest module can no longer move without detaching, the the next closest moves, etc., until the goal object is enveloped. In this way, the goal is an attractor for modules, but kinematic constraints keep the robot in one piece. Figure 9 shows the evolution of configurations generated using this approach. Local heuristic algorithms for the motion planning problem (which incorporate the kinematic information developed here) are developed and compared in the tech report [SMC94].

5 Conclusions

The concept and kinematics of a metamorphic robotic system was developed in this paper. Potential applications, design problems, and algorithms for task implementation were presented. It was shown how simple rules can be used to enforce rather complex (and useful) behavior. Currently, a prototype metamorphic robotic manipulator is under development.

6 Acknowledgements

This work was made possible by New Young Investigator Award IRI-9357738 from the Robotics and Machine Intelligence Program at the National Science Foundation.

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