Proceedings of the 1996 IEEE
International Conference on Robotics and Automation
Minneapolis, Minnesota - April 1996

Bounds for Self-Reconfiguration of Metamorphic Robots

Gregory Chirikjian   Amit Pamecha
Department of Mechanical Engineering
The Johns Hopkins University
Baltimore, MD 21218

Abstract

A metamorphic robotic system is a collection of mechatronic modules, each of which has the ability to connect, disconnect, and climb over adjacent modules. A change in the macroscopic morphology results from the locomotion of each module over its neighbors. In this paper, lower and upper bounds are established for the minimal number of moves needed to change such systems from any initial to any final specified configuration. These bounds are functions of initial and final configuration geometry and can be computed very quickly, while solving for the precise number of minimal moves cannot be done in polynomial time. These bounds can be used to 'weed out' and improve inefficient reconfiguration strategies, and provide a benchmark for the evaluation of heuristics in general.

1 Introduction

A *metamorphic* robotic system is a collection of independently controlled mechatronic modules, each of which has the ability to connect, disconnect, and climb over adjacent modules . Each module allows power and information to flow through itself and to its neighbors. A change in the metamorphic robot topology (i.e., a change in the relative location of modules within the collection) results from the locomotion of each module over its neighbors. Thus a metamorphic system has the ability to dynamically self-reconfigure. Changes in configuration with the same topology are achieved by changing joint angles, as is the case for standard (fixed-topology) robotic manipulators.

What distinguishes metamorphic systems from other reconfigurable robots is that they possess all of the following properties: (1) All modules have the same physical structure, and each must have complete computational and communication functionality; (2) Symmetries in the mechanical structure of the modules must be such that they fill planar and spatial regions with minimal gaps; (3) The modules must each be kinematically sufficient with respect to the task of locomotion, i.e., they must have enough degrees of freedom to be able to 'walk' over adjacent modules so that they can reconfigure without outside help; (4) Modules must adhere to adjacent modules, e.g., there must be connectors between modules which can carry load.

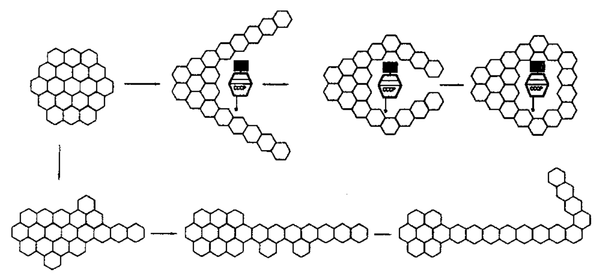

Figure 1: An Example Metamorphic Robot with Applications

While a wide variety of module designs satisfy the above conditions, one particular class is discussed here. These modules are mechanisms which can be represented as polyhedra, e.g., certain kinds of platform manipulators in the spatial case, or closed linkages in the plane. Figure 1 shows a planar example where the modules are six bar linkages. Hardware implementations that satisfy the above conditions can be found in [MuKK94,Yim94,PKCh95].

Potential applications of metamorphic systems include : Obstacle avoidance in highly constrained and unstructured environments; 'Growing' structures composed of modules to form bridges, buttresses, and other civil structures in times of emergency; Envelopment of objects, such as recovering satellites from space; Micro-robots for medical applications. Some of these applications are shown in Figure 1.

This paper addresses issues in the motion planning of metamorphic systems with a fixed base, i.e., 'manipulators,' as opposed to 'mobile robots.' In Section 2, we formalize the motion planning/reconfiguration problem. In Section 3 and 4 we establish upper and lower bounds respectively on the number of moves required to reconfigure between any given initial and final configurations. For a review of pertinent literature, see [Ch94, ChPE96].

2 General Formulation of the Motion Planning Problem

In this section we formalize the motion planning problem for metamorphic robotic systems. Figure 2 shows a schematic and hardware demonstration of a

0-7803-2988-4/96 $4.00 © 1996 IEEE

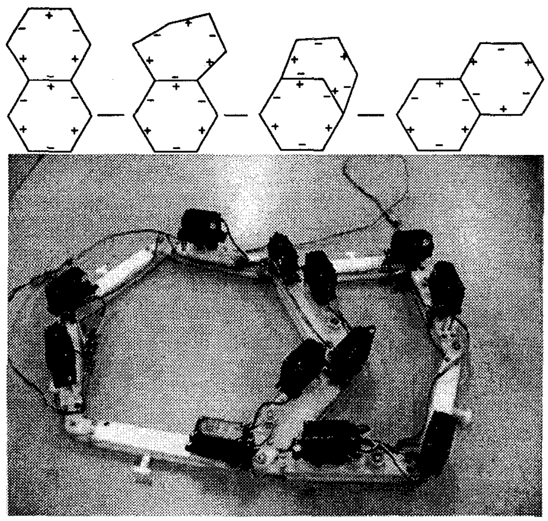

Figure 2: Schematic and Hardware example of Module Motions

single planar module locomoting over another module. For further demonstration of this type of locomotion with hardware, see [PaCh95, MuKK94]. The kinematic constraints governing the motion of one module over the surface of a collection of other modules are:

- Modules can only move into spaces which are not already occupied by other modules.
- Every module must remain connected to at least one other module, and at least one of the modules must stay connected to the fixed base from which the collection of modules originated.
- A single module may only move one lattice space per timestep, and it achieves this motion by deforming and mating faces to faces (or in the planar case edges to edges).

Under these constraints, the motion planning/self-reconfiguration problem is stated as : *the determination of the sequence of module motions from any given initial configuration to any given final configuration in a reasonable (preferably minimal) number of moves.* Factors which complicate this are (1) module motions do not commute; (2) modules are very restricted in their movements. This is because in many situations module motions cannot be composed, i.e., the motion of one module prevents any allowable motion of a neighboring module.

Motion planning/self-reconfiguration cannot be achieved by algorithmic search methods for a large number of modules because the computational complexity of this approach is too great. This is due to the fact that for a large number of modules, the number of possible robot configurations is huge. In fact, a number of works have dealt with the enumeration of configurations composed of modules [ChB93,Go65,HaP73,HaR70,Lu72], and the general problem is to the best of our knowledge still unsolved. For example, in the simplified case dealt with in [HaR70] for hexagonal cells (modules), the number of configurations generated (N) is asymptotic to

$$N = \frac{(2n-1)!}{(n-1)!(n+1)!}\left(\frac{5}{4}\right)\sqrt{5}, \qquad (1)$$

where n is the number of modules.

This, as well as the results of other works, suggests a very rapid (nonpolynomial) growth in the number of different configurations as a function of the number of modules. However, establishing the *number* of different configurations consisting of n modules is just the first part of the problem. If each of these N configurations represent a vertex on a graph, *finding* the shortest path on this graph (optimal reconfiguration sequence) takes $O(N^2)$ computations [Ak87,ReND77]. However, since N grows so rapidly as a function of n this approach is not practical for large n.

Since an optimal sequence of module motions cannot be found efficiently, we desire to bound from below and above the minimal number of moves required to get from one connected configuration to any other with the same number of modules. This gives us a tool to evaluate and/or improve the performance of any given heuristic.

We also desire that these bounds have the following properties:

- They can be computed quickly.
- They are a function of easily quantified characteristics of the initial and final configurations, e.g., geometric parameters such as perimeter, area, moments of area, intersections, unions, etc.
- They should couple our concepts of distance between modules/lattice points and number of moves between configurations.
- They should be tight bounds.

The next two sections establish bounds that satisfy the above conditions.

3 Upper Bounds on Minimal Number of Module Motions

In this subsection, we derive closed-form upper bounds on the minimal number of module moves required to reconfigure between arbitrary configurations. These upper bounds are functions of the initial and final 'perimeters' of the configurations, and the largest possible perimeter that a connected configuration of n modules can have. In addition, these bounds are functions of the number of modules in the largest simply connected overlap between the two configurations, I'_{AB}. We begin by formalizing some intuitive concepts.

Definition : The 'exterior' of a configuration (collection of modules) is the union of all lattice spaces not within the configuration of modules, but at most one lattice space distant from a module in the configuration.

Definition : The 'perimeter,' $P(n)$, of a configuration is the *number* of lattice spaces in the exterior of the configuration.

Definition : A 'movable module' is a module that can move from its current lattice space to an adjacent space in the exterior of the current configuration without isolating itself or disconnecting any other modules in the process.

Definition : A 'maximal simply-connected overlap,' I'_{AB}, of two configurations A and B is the largest simply connected subset of modules contained in $A \cap B$ which contains the base module. (This subset need not be unique, but the number of modules in any such subset is maximal.)

In the case of planar modules, the above definition of perimeter reduces to the sum of the minimal number of empty spaces that surround the collection of modules together with the minimal number of modules in the interior of each 'hole' if the configuration has loops. For planar illustrations of each of the above definitions, see Figure 3.

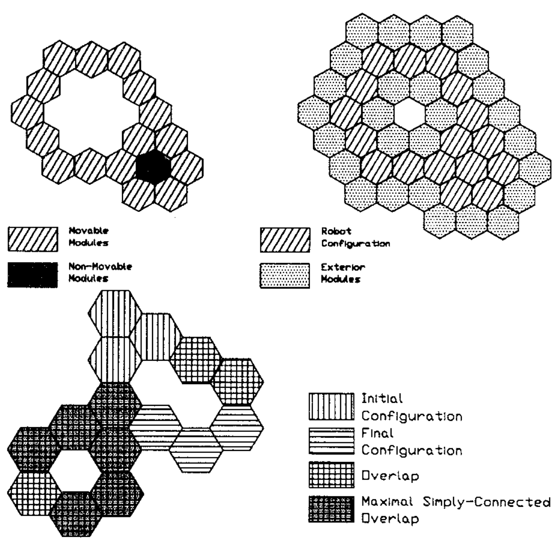

Figure 3: Movable Modules, Exterior Modules and Maximal Simply-Connected Overlap

We are now armed with the major definitions needed to derive an upper bound on the minimal number of moves required to reconfigure from any initial configuration to any other with the same number of modules.

There are three perimeters that will be of particular interest to us for a connected configuration consisting of n modules: (1) the perimeter of an $n-1$ module connected subset of the initial configuration (where one movable module has been removed such that the resulting perimeter is minimized): $P_i(n-1)$; (2) the perimeter of an $n-1$ module connected subset of the final configuration (where one movable module has been removed such that the resulting perimeter is minimized): $P_f(n-1)$; and (3) the greatest perimeter of any connected configuration of $n-1$ modules : $P_{max}(n-1)$. For systems composed of planar hexagonal modules $P_{max}(k) = 2(k+2)$, and this value occurs for serial configurations without branches or loops. For the sake of notational compactness, we will sometimes refer to the above functions without their arguments when there is no ambiguity.

3.1 A Simple Upper Bound

Theorem: An upper bound on the minimal number of moves required to reconfigure between any two configurations A and B with n modules and maximal simply connected overlap with I'_{AB} modules is: $U_1(A,B) = (n - I'_{AB})(P_{max}(n-1))/2$.

Proof: When a given module moves, it is not counted in the perimeter it must traverse. By definition, there is no path that a module can take in the exterior of a configuration which is longer than the full perimeter. Thus, if a module takes a path to, and returns from, any arbitrary lattice space in the exterior of the configuration, it will take at most $P_{max}(n-1)$ moves because it is traversing a configuration with $n-1$ modules. Any minimal length path connecting two different points will thus be at most half of this length, because either the circuit has equal length on outgoing and return paths, or else we can always choose the smaller one. This process is repeated the fewest number of times needed to reconfigure (for the tightest bound). This number is the number of modules not in a maximal simply connected overlap of the two configurations (which is $n - I'_{AB}$). Hence the maximum number of moves required are $(n - I'_{AB})(P_{max}(n-1))/2$.

We impose the restriction that the largest simply connected overlapping region including the base (as opposed to the whole overlap) need not move for the following reason. If the overlap is not simply connected, modules from one configuration might be inside a 'hole,' while modules of the other configuration could be on the outside (see Figure 4). Similarly, if the overlap is not connected at all, there may be no way to reconfigure without moving the overlapping modules. ∎

3.2 A Tighter Upper Bound

While the bound discussed in the previous section is a valid upper bound, it can be made tighter by incorporating information which is readily available, i.e., the initial and final perimeters. Let M denote the maximal amount of change which the motion of one module can make to the perimeter of a configuration. Furthermore, let us choose the initial and final perimeters traversable by a module to be the smallest of all possible perimeters of $n-1$ connected modules contained in the initial and final configurations, respectively. We can do this without loss of generality

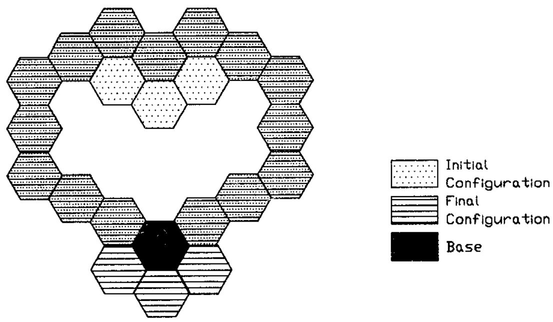

Figure 4: An Example of Non-Simply Connected Overlap

by examining all modules which are able to move in the initial and final configurations, and choosing the first and last modules so that the perimeter created by excluding these modules is minimal. The following theorem incorporates all this information:

Theorem: A tighter upper bound on the minimal number of moves required to reconfigure between any two configurations with (1) n modules; (2) maximal simply connected overlap consisting of I'_{AB} modules; (3) initial, final, and maximal possible perimeters P_i, P_f, and P_{max}; is the lesser of the following expressions:

$$\begin{aligned} U_2(A,B) = [P_i i_1 + M i_1(i_1-1)/2 + P_{max}(i_2-i_1) \\ +(n-I'_{AB}-i_2)(P_f+nM-I'_{AB}M) \qquad (2) \\ -M(n-I'_{AB})(n-I'_{AB}-1)/2 + Mi_2(i_2-1)/2]/2 \end{aligned}$$

and

$$\begin{aligned} U_2(A,B) = [P_i i_3 + M i_3(i_3-1)/2 \\ +(P_f+nM-I'_{AB}M)(n-I'_{AB}-i_3) \qquad (3) \\ -M(n-I'_{AB}-1)(n-I'_{AB})/2 + Mi_3(i_3-1)/2]/2 \end{aligned}$$

where i_1, i_2 and i_3 are integers defined by the expressions: $P_i + i_1 M = P_{max}$; $P_{max} = P_f + (n - I'_{AB} - i_2)M$, and $P_i + i_3 M = P_f + (n - I'_{AB} - i_3)M$. M is the largest amount by which the perimeter of a configuration can *change* (increase or decrease) by the motion of one module (in the hex case, $M = 4$).

Proof: Let $p(j)$ be an upper bound on the number of moves required for the j^{th} module to move from its initial position to the final position. The half perimeter that the first module traverses is bounded from above by $p(0) = P_i/2$, the second will be bounded from above by $p(1) = (P_i + M)/2$, and the $j+1^{st}$ will be bounded from above by $p(j) = (P_i + jM)/2$, until j is large enough that either $P_i + i_1 M = P_{max}$, or $P_i + i_3 M = P_f + (n - I'_{AB} - i_3)M$ for some integers $j = i_1$ or $j = i_3$. That is, until the perimeter reaches its maximal possible value, or it reaches such a value that the perimeter must start to decrease in order to attain the perimeter of the final configuration with moves of the remaining modules. The above conditions have the geometric meaning of where the lines defined by $p(j) = P_i + jM$, $p(j) = P_{max}$, and $p(j) = P_f + (n - I'_{AB} - j)M$ intersect in the plane whose independent coordinate is j and dependent coordinate is p as shown in Figure 5. In other words, the lines with slope $\pm M$ will intersect each other either above the line with zero slope or below it. This in turn depends on the initial and final perimeters (the intercepts of the lines) and M (the slope of the lines). Since the moves associated with each half perimeter are added to the total, and we seek a bound on the minimal total moves, we seek the planar figure that will have the least area bounded by these three lines and the j axis. This will either be a triangle with peak below P_{max}, or a trapezoid with P_{max} as the top line. The expressions in the statement of the theorem correspond to these cases, and are derived below. The two cases are depicted in Figure 5.

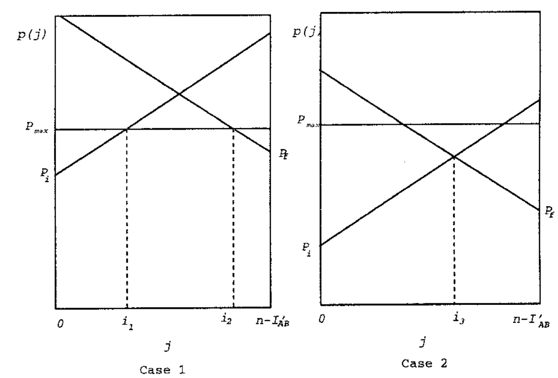

Figure 5: Graphical Derivation of Upper Bounds

(case 1) Trapezoid: By definition, i_1 is where the line with slope $+M$ intersects the horizontal line at P_{max}. Starting at $j = i_1$ the half perimeter traversed will be at most $P_{max}/2$ (since this is the maximal value possible). This is true until $P_{max} = P_f + M(n - I'_{AB} - i_2)$, for some integer $j = i_2$. This is where the line with slope $-M$ intersects the horizontal line at P_{max}. From $j = i_2$ on, the perimeter must decrease in the steepest way possible to be able to reach P_f using the unmoved modules. The perimeter for the remaining moves will be bounded from above by $p(j) = P_f + M(n - I'_{AB} - j)$ in order for it to be possible to attain the final perimeter. If we sum up all three contributions from $j = 0$ to $n - I'_{AB} - 1$, case 1 is proved. In other words, the upper bound is:

$$\sum_{j=0}^{n-I'_{AB}-1} p(j) =$$

$$\sum_{j=0}^{i_1-1}(P_i+jM)+\sum_{j=i_1}^{i_2-1}P_{max}+\sum_{j=i_2}^{n-I'_{AB}-1}(P_f+(n-I'_{AB}-j)M).$$

These summations simplify using the formulas: $\sum_{j=k}^{r}1 = r-k+1$, $\sum_{j=k}^{r} j = r(r+1)/2 - k(k-1)/2$, yielding Equation (2).

(case 2) Triangle: Suppose that the lines given by the equations $p(j) = P_i + jM$ and $p(j) = P_f + (n - I'_{AB} - j)M$ intersect each other below the line $p(j) = P_{max}$. By denoting as i_3 the value of j where these lines intersect,the perimeter is computed by summing along the first line until i_3, and then switching to the second line. The result of this summation is:

$$\sum_{j=0}^{n-I'_{AB}-1} p(j) =$$

$$\sum_{j=0}^{i_3-1}(P_i+jM)+\sum_{j=i_3}^{n-I'_{AB}-1}(P_f+(n-I'_{AB}-j)M),$$

which is simplified using the same formulae as case 1, yielding Equation (3). ∎

Though the expressions for the bounds described above appear complicated, they have the advantage that they can be computed in $O(1)$ calculations. They can also tell us right away that there will be at most $O(nP_{max})$ moves required to reconfigure *no matter what module design is chosen.* In the case of planar modules this will be $O(n^2)$. However, in practice we would like to have *constructive* upper bounds on the minimal number of moves. That is, instead of conservatively assuming that half of the largest perimeter is traversed each time a single module moves to fill a space in the new configuration, we can construct intermediate configurations by having modules move along the perimeter until they stop at a suitable place in the desired configuration. This will, by definition require a fewer number of moves than the nonconstructive upper bounds presented above and will run in at most $O(n^2)$ calculations in the planar case.

4 Lower Bound on Minimal Number of Module Motions

A good lower bound on the minimal number of moves required to reconfigure a metamorphic robot is obtained by using the lattice metric and concepts of optimal assignment. The lower bound presented here is based on the fact that the minimal number of moves required for a single module motion in a lattice will be no less than the lattice distance, δ_L, between the initial and final spaces [PCh96]. If it were possible to track the sequence of motions of an optimally reconfiguring metamorphic robot, we could compute the lattice distance between each module in its initial and final lattice spaces, and the sum would be a lower bound on the total number of module motions. Since this is not possible, we will assign modules in two configurations in such a way that the sum of the lattice distances between matched modules is minimized over all possible matchings. This minimal sum will be at most the aforementioned lower bound. Since this *is* something that can be computed relatively efficiently (in at most $O(n^3)$ computations, see [PCh96] for details), this is the lower bound we will use. The basic approach is summarized below.

Let the present configuration of the robot be described by the set of modules A, where $a_i \in A$ represents a module in configuration A for $i = 1, ..., n$. Let the new configuration be defined by the set B , where $b_j \in B$ for $j = 1, ..., n$ represents a module in configuration B. A lower bound on the total number of moves required to go form A to B (or vice versa) is given by an optimal assignment of each element a_i in A to an element b_j in B, $f : A \rightarrow B$, such that the sum of the distances (as defined by the lattice metric) for the assignment is minimized. Equivalently, this can also be treated as finding a perfect matching in a weighted *bipartite graph* $G = (A, B)$, such that the sum of the weights of the matching is minimized. We call this $L(A, B)$.

5 An Illustrative Example

In this section, a planar metamorphic system with hexagonal modules is used to demonstrate the methods developed in the previous sections.

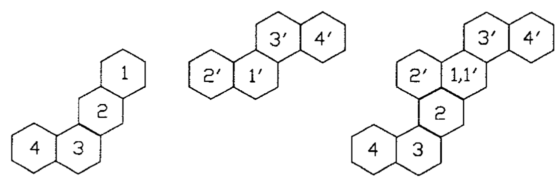

Figure 6: Example Initial and Final Configurations

For an illustration of the lower bound on minimal number of moves, consider the following example. Figure 6(a) shows the present configuration, Figure 6(b) shows the new configuration and Figure 6(c) shows an arbitrary labeling of the modules in the two configurations. Out of all possible matchings of the labels $\{1, 2, 3, 4\}$ with $\{1', 2', 3', 4'\}$ (of which there are $4! = 24$), we choose one for which the sum of the lattice distances between matched modules is minimized. In this case it is easy to see that $L(A, B) = 8$. The reader is encouraged to verify this by trying all possible ways of bijectively matching modules and summing the lattice distances. One such optimal assignment in this case results from matching like numbered modules in Figure 6, i.e., $i \rightarrow i'$, and summing the lattice distances between all of them. It should be noted that ennumerating all possible matchings is very inefficient ($O(n!)$). $O(n^3)$ algorithms for optimal assignment are used in practice.

Now let us consider the upper bounds on the minimal number of moves computed for this configuration.

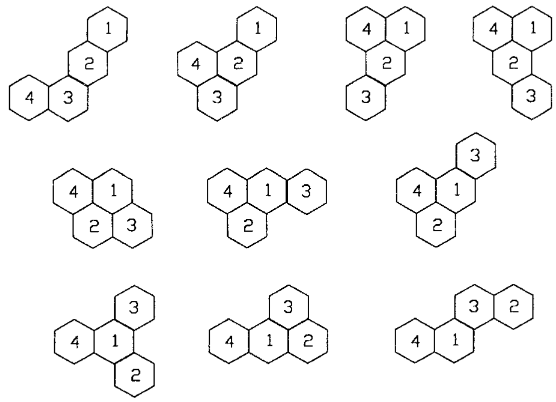

Figure 7: An Optimal Reconfiguration Sequence

In this particular case, $P_f(3) = P_i(3) = P_{max}(3) = 10$. The closed-form upper bounds are both computed simply as: $3 \cdot 5 = 15$. Constructively computing an upper bound by rolling 4 to $4'$, 3 to $3'$, and 2 to $2'$, one gets 9. Thus, $8 \leq M_{min} \leq 9 < 15$ for this example. In fact, for such a small number of modules, it is easy to test all possible combinations of moves by hand, and one finds that $M_{min} = 9$. Figure 7 explicitly represents one possible strategy for optimal reconfiguration. The initial labels have been retained so that motions are easy to track.

For large number of modules, heuristic searches would have to be used with the bounds presented here guiding the search.This issue is discussed in greater detail in [ChPE96].

6 Conclusions

In this paper bounds on the fewest moves required to reconfigure from one configuration of a metamorphic robot to another were established. These bounds are important because explicit solution for the minimal number of moves becomes computationally infeasible when the number of modules is greater than ten. In addition to providing a bechmark for testing heuristic algorithms, these bounds can be used to 'weed out' and improve inefficient motion planning strategies. Furthermore, the concepts developed here provide a framework from which efficient heuristics can be constructed.

7 References

[Ak87] Akman, V., *Unobstructed Shortest Paths in Polyhedral Environments*, Lecture Notes in Computer Science, 251, Springer-Verlag, New York, 1987.

[ChB93] Chen, I.-M., Burdick, J.W., "Enumerating Non-Isomorphic Assembly Configurations of a Modular Robotic System," Proceedings of *IEEE/RSJ International Conference on Intelligent Robots and Systems*, pp 1985-1992, Yokohama, Japan, July, 1993.

[Ch94] Chirikjian, G.S., "Kinematics of a Metamorphic Robotic System," *Proceedings of the 1994 IEEE Int. Conf. on Robotics and Automation*, San Diego, CA, May 1994, pp.449-455.

[ChPE96] Chirikjian, G.S., Pamecha, A., Ebert--Uphoff,I., "Evaluating Efficiency of Self-Reconfiguration in a Class of Modular Robots," *Journal of Robotic Systems,* 1996 (To appear).

[FuK90] Fukuda, T., Kawauchi, Y., "Cellular Robotic System (CEBOT) as One of the Realization of Self-organizing Intelligent Universal Manipulator," *Proc. 1990 IEEE Conf. on Robotics and Automation*, pp. 662-667.

[Go65] Golomb, S.W., *Polyominoes,* Scribner, New York, 1965.

[HaP73] Harary, F., Palmer, E.M., *Graphical Enumeration,* Academic Press, New York, 1973

[HaR70] Harary, F., Read, R.C., "The Enumeration of Tree-like Polyhexes," *Proc. Edinburgh Math. Soc.,* Vol. 17, 1970, pp. 1-13.

[Lu72] Lunnon, W.F., "Counting Hexagonal and Triangular Polyominoes," *Graph Theory and Computing* (R.C. Read, ed.), Academic Press, New York, 1972, pp. 87-108.

[MuKK94] Murata, S., Kurokawa, H., Kokaji, S., 'Self-Assembling Machine," *Proceedings of the 1994 IEEE Int. Conf. on Robotics and Automation*, San Diego, CA, May 1994, pp.441-448.

[PCh95] Pamecha, A., Chirikjian, G.S., "A Metamorphic Robotic System: Simulation and Connector Mechanism Demonstration" *Video Proceedings, 1995 IEEE Int. Conf. on Robotics and Automation* Nagoya, Japan, May 1995.

[PCh96] Pamecha, A., Chirikjian, G.S., "A Useful Metric for Modular Robot Motion Planning," *1996 IEEE Int. Conf. on Robotics and Automation* Minneapolis, USA, April 1996.

[PKCh95] Pamecha, A., Kohaya, Y., Chirikjian, G.S., "Design of Error Tolerant Coupling Mechanisms for Metamorphic Robots," *Third IASTED Conference on Robotics and Manufacturing*, Cancun, Mexico, June 1995, pp 7-13.

[ReND77] Reingold, E.M., Nievergelt, J., Deo, N., *Combinatorial Algorihms, Theory amd Practice,* Prentice Hall, Englewood Cliffs, New Jersey, 1977.

[Ro84] Roberts, F., *Applied Combinatorics,* Prentice Hall, Englewood Cliffs, New Jersey, 1984.

[Yim94] Yim, M.,"New Locomotion Gaits," *Proceedings of the 1994 IEEE Int. Conf. on Robotics and Automation*, San Diego, CA, May 1994.