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# A New Potential Field Method for Robot Path Planning

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## Abstract

*This paper presents a new artificial potential field method for path planning of non-spherical single-body robots. The model simulates steady-state heat transfer with variable thermal conductivity. The optimal path problem is then the same as a heat flow with minimal thermal resistance. The novelty of this technique is that the thermal resistance in the configuration space for all different orientations of the robot can be superimposed. This reduces a search on $R^n \times SO(n)$ to one on $R^n$ followed by a search on $SO(n)$. Examples are presented to demonstrate the approach.*

## 1. Introduction

The artificial potential field is a useful tool in path planning. The main idea is to construct an attractive potential at the goal, and repulsive potentials on the obstacles. The path is then generated by following the gradient of a weighted sum of potentials. Numerous artificial potential functions have been proposed in the past decade [barraquand91, canny90, Khatib86, khosla88]. The usual formulations of potential fields suffer from local minima, which cause the robot to stop at unintended locations. This limits the applicability of the artificial potential approach. Looking for a potential field without local minima has become a central concern in this approach.

The navigation function proposed by Rimon and Koditschek [rimon92] is free from local minima for limited classes of object shapes and configuration spaces. At the cost of increased computational complexity, more general environments can be constructed using diffeomorphic mappings. Sundar and Shiller [sundar97] approach the problem by establishing a potential field using Hamilton-Jacobi-Bellman (HJB) theory. They formulate the path by solving the HJB equation. It is computationally difficult for a large number of obstacles, even for a simple case such as circles.

Attracted by the elegant properties of harmonic functions (solutions of Laplace Equation), some recent research has focused on the use of harmonic functions as potentials [connolly90, connolly93, connolly97, guldner93, guldner97, keymeulen94, kim91, tarassenko91]. Harmonic potentials completely eliminate local minima by nature since they satisfy a min-max principle. Quite similar to the harmonic potential method, Masoud *et. al.* [masoud94] establish a biharmonic potential field which originated in solid mechanics. Schmidt and Azarm [schmidt92] build a potential field using an unsteady diffusion equation. The disadvantage of using the transient state equation is its slow response to a dynamic environment and time-delays. All of these potential methods, however, address only the obstacle avoidance problem with no concern for path optimality and robot orientation.

We propose a new artificial potential method based on heat transfer with variable thermal conductivity, $K$. The obstacles are described by very low $K$ materials while free-space is assumed to have very high $K$ materials (see Figure 1). A convenient feature of using variable $K$ is that it enables us to use a simple rectangular grid to dis-

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cretize a workspace. Therefore, not only can we get a simple computational domain which allows us to use fast solvers, but also save time for grid generation which is required by the Laplace equation approach when the obstacle shapes are irregular. Furthermore, the arbitrary shape and changes in the obstacles can be easily reflected by simply adjusting the thermal conductivity function while the grids have to be regenerated for other methods that use boundary conditions to describe obstacles. Adopting variable $K$ to describe obstacles and free-space is much more suitable for arbitrarily shaped objects and is useful for dynamic environments.

The format of the remainder of this paper is as follows. In section II, our approach is described. A strategy for non-spherical robots and orientation information is presented in Section III. Section IV provides computer simulation examples.

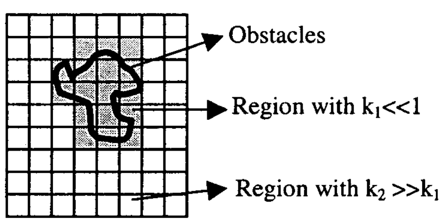



Figure 1. Workspace description using variable thermal conductivity $K$. $k_1$ is very low $K$ and $k_2$ is very high $K$.

# 2. The Proposed Approach

## 2.1 The Heat Transfer Analogy

Different physical analogies have been employed in the past to describe artificial potential fields. Examples are electrostatics [guldner93, guldner97, tarassenko91], incompressible fluids dynamics [keymeulen94, kim91], gaseous substance diffusion [schmidt92] and mechanical stress [masoud94].

We formulate path planning as a steady-state heat transfer problem. The reason we use heat transfer is because of the features of heat flux and thermal conductivity. Heat flux is vectorial in the sense that it points in the direction of a negative temperature gradient. There is a monotonically decreasing temperature along a path between the sink and an arbitrary point within the temperature field. Heat always flows into a sink and never enters region with zero thermal conductivity by nature.

Steady-state heat transfer is independent of time. It allows the model of the environment to be updated incrementally and has fast response to changes in the environment. Therefore, dynamic environments can be modified on-line during execution.

In our heat transfer analogy, the goal is treated as a sink pulling heat in. The obstacles are indicated by zero (or very low) thermal conductivity. According to the tasks and environment, the sources are either assigned to the robots or to discrete nodes in the free-space. As the result of a heat conduction process, a temperature distribution develops and the heat flux lines that are flowing to the sink fill the workspace. Such a field can be seen as a communication medium among the goal, robots and obstacles. The path can be easily found by following the heat flux.

## 2.2 Potential Field Design

The temperature is utilized as the artificial potential field to specify the desired paths. The temperature is characterized by its harmonic property. Hence, it is free from local minima. In the model, steady-state heat conduction with internal heat sources, sink, and variable thermal conductivity are built. Thermal insulated boundary conditions are considered. The resulting mathematical model is:

$$\nabla \bullet (K \nabla T) = q \qquad (1)$$

$$\int_{\Omega} q \mathrm{dV} = 0 \qquad (2)$$

$$\left(\frac{\partial T}{\partial n}\right)_\Gamma = 0 \qquad (3)$$

$$\underline{f} = -K\,\nabla T \qquad (4)$$

$T$ denotes the temperature over the workspace. $q$ indicates the heat sources and sink. $K$ is the thermal conductivity which is a function of space coordinates. $\Omega \subset R^n$ defines the configuration space of a translating robot, and $\Gamma \subset R^n$ the boundary of the C-space. $\nabla$ represents the grad operator and $\nabla\bullet$ represents the divergence operator. $n$ expresses the unit normal vector. $f$ is the heat flux.

Numerical solutions to the above mathematical model are readily obtained from finite difference methods or finite element methods.

### *2.3 Optimization Algorithm*

The source points are the singularity points. It is not possible to start exactly at a source point. A start position can be selected as a point on the arc of a circle that is a small distance away from the source. However, there exist many candidate paths due to the choice of different initial directions around the circle. Using the concept of thermal resistance, we developed a new analytical method for path optimization.

Thermal resistance for conduction, $R_t$, is associated with the thermal conductivity inversely and the length of the heat flow path directly. The optimal robot path should have the shortest length without traversing the obstacle regions. Hence, we want a heat flux line with shortest heat flow path and highest thermal conductivity. In other words, the lower the thermal resistance is, the shorter the path is and the less the opportunities to meet obstacles. The path optimization problem can be viewed as looking for the heat flux with the minimal thermal resistance. The analytical optimization equation is derived as:

$$R_t = C\int_0^L \frac{1}{K(\underline{x})^w}\,dL \qquad (5)$$

Where C is a constant used to scale the value of thermal resistance, $w$ is a parameter used to emphasis the effect of thermal conductivity $K$, and $L$ is the length along heat flow path.

## 3. The Strategy for Orientation

Despite the vast number of artificial potential methods proposed in the literature, few works discuss the orientation of a non-spherical robot in path planning. They usually escape the orientation problem by pointing out that a non-spherical robot can be transformed into a point robot by expanding the obstacles by the largest radius or maximum size of the robot. This significantly reduces and wastes the available free-space and can not be used to obtain an orientation path.

One technique that does take into account orientation is to convolve the shape of a non-spherical robot at each orientation with the obstacles [kavraki95], and then do a search in $R^n \times SO(n)$. The high dimensionality of the space can be a problem.

We develop a novel strategy for compressing the orientations so that a search in $R^n \times SO(n)$ is reduced to a search in $R^n$ followed by one in SO(n). Generally speaking, there are two main steps. For a clear explanation, the 2D case is stated as follows.

**Step 1. Construct the CS.**

First, the robot and each obstacle are convolved at every orientation, *i.e.*, from 0 to $2\pi$ as in [kavraki95]. The convolved results are planner geometric shape in parallel with the x-y plane. Then, the convolved results with a given height are piled up from 0 to $2\pi$ sequentially along the Z-axis. The maximum value of the Z-axis is no more than $2\pi$ due to the periodic nature. This three-dimensional space, C-space, is $R^2 \times [0, 2\pi)$. Thus the obstacles in the workspace have been transformed to the configuration space obstacles (CS-obstacle) and a non-spherical robot in $R^2$ can be viewed as a point robot in the C-space. Figure 2 shows an example of a CS-obstacle resulting from a 5×10 rectangular robot and a 5×5 square obstacle.

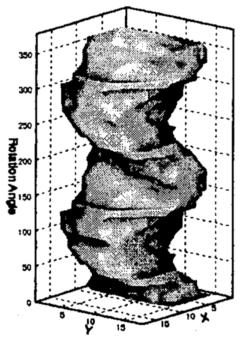


Figure 2. A CS-obstacle resulting from a robot (5x10) and an obstacle (5x5).

**Step 2. Generate path in C-space**

This can be approached in two ways. First, if the C-space is low dimensional, we can do the path planning in the same way as that for a point robot in 3D space. The path in C-space is generated using the proposed methods stated in Section 2. Second, if the C-space is high dimensional (such as $R^3 \times SO(3)$ ) we can add together the thermal resistances of each C-space slice defined by fixed robot orientation. Then we perform a translational heat diffusion problem to find a translational path in this reduced C-space. Finally we do a gradient search on orientation after this translational path of least resistance is found.

## 4. Simulation

In a static environment, the goal and the obstacles are fixed. Once the potential field has been generated, it can be reused as often as desired. The path generation proceeds very fast, since it only involves evaluation of the heat flux over a known temperature potential.

Path planning in maze-like obstacles is shown in Figure 3. The smile face indicates the goal position and the circle the start point of the robot. The paths generated from all the different positions utilize the same temperature potentials. Figure 4 illustrates path planing in 3D space. The proposed path optimization algorithm is verified by the example given in Figure 5. The obstacles have a very low thermal conductivity and the free-space a very high thermal conductivity. Eleven paths are shown which begin at the start point and end at the goal. The number beside each path is the thermal resistance calculated by Equation 6. The parameter $w$ is set to 1 in this case. It is seen that paths that traverse the obstacles have higher thermal resistance though the length of the path looks short. The paths in free-space have the lower thermal resistance. Among these paths, the lowest thermal resistance is 9, which is highlighted by a star circle. This path is the optimal path.

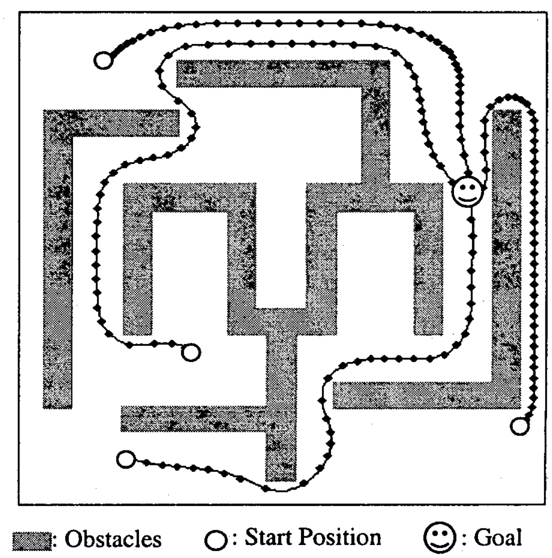
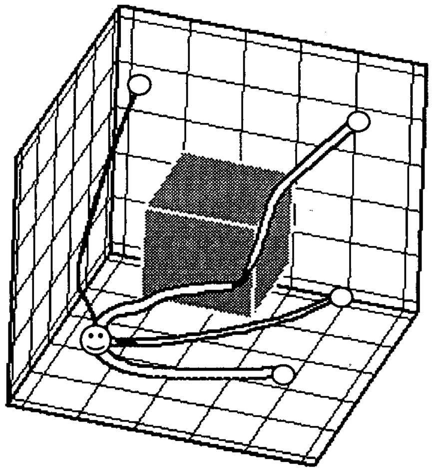



Figure 3. Paths generated in a maze-like obstacle.

Figure 6 shows the obstacle field with an L-shaped robot. The initial and final configurations are bold and the path is generated by first compressing C-space thermal resistance to $R^2$, then solving a heat diffusion problem followed by a search over orientations. Figure 7 shows the superimposed conductivity. Dark means low conductivity. We note that if the obstacles had been considered impermeable and the C-space constant-theta slices are added, then no path through the obstacles would exist.



: Obstacles O: Start Position ☺: Goal

Figure 4. Path planing in 3D space

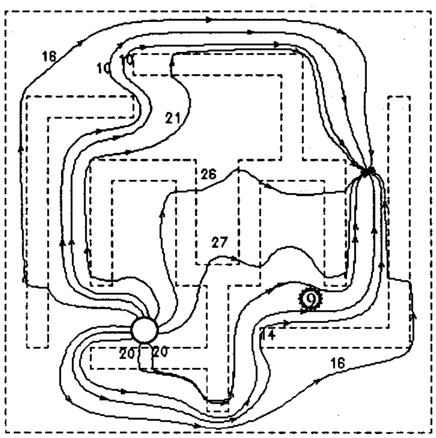



Figure 5. Verification of the path optimization method

## 5. Conclusions

This paper presents a new addition to the family of artificial-potential-based path-planning methods. The new approach is motivated by steady-state heat transfer with variable thermal conductivity. Variable thermal conductivity is used to describe the obstacles and free-space. The advantages of using variable thermal conductivity are that it allows for a simple geometrical domain regardless of obstacle complexity and can handle changes in the environment. Borrowing the concept of thermal resistance, a simple path optimization method is derived. This paper also proposes a novel strategy for non-spherical robot path planing, especially for compressing down the orientational part of the C-space to reduce the complexity of the path planning problem into a sequential translation-rotation search.

The path generated by the proposed approach has various favorable features such as freedom from local minima, smoothness, optimality and collision-avoidable. This approach can be applied to rich classes of robot path planning in both static and dynamic environments.

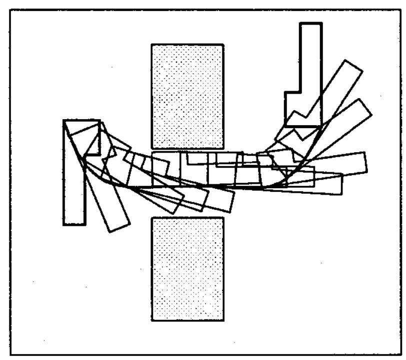


Figure 6. Non-spherical robot path planning

Figure 7. C-space conductivity integrated over robot orientation

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