New Probabilistic Approaches to the AX = XB Hand-Eye Calibration without Correspondence

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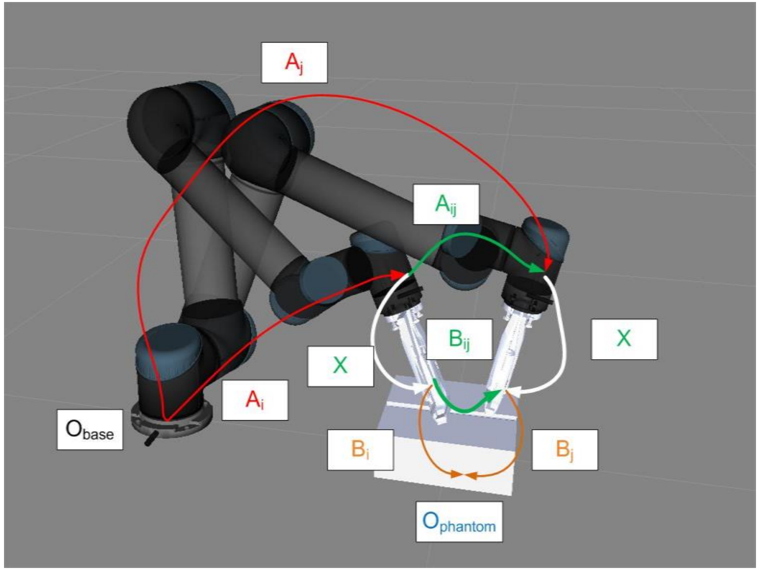
Problem of Interest

- Hand-eye (wrist-camera) calibration,
  Ultrasound probes,
  Aerial vehicle sensors
- At least two compatible measurements $\{A_1, B_1\}$ and $\{A_2, B_2\}$ are needed
- Most of the algorithms assume exact correspondence
- We propose new probabilistic methods that don't require a priori knowledge of correspondence



The AX = XB in a Ultrasound Sensor Calibration Setup

Mathematical Background

$$A_i X = X B_i$$

$$\Downarrow$$

$$(f_A * \delta_X)(H) = (\delta_X * f_B)(H)$$

$$\Downarrow$$

$$\boxed{M_A\, X = X\, M_B}$$

$$\boxed{Ad(X^{-1})\, \Sigma_A\, Ad^T(X^{-1}) = \Sigma_B}$$

(1) Mean Equation

(2) Covariance Equation

Integration + Convolution on SE(3)

Integration

$$\int_{SE(3)} f(H)dH \doteq \int_{\mathbf{q}\in D} f(H(\mathbf{q}))|J(\mathbf{q})|d\mathbf{q}$$

$$J(\mathbf{q}) = \left[\left(H^{-1}\frac{\partial H}{\partial q_1}\right)^{\vee}; \left(H^{-1}\frac{\partial H}{\partial q_2}\right)^{\vee}; \cdots \left(H^{-1}\frac{\partial H}{\partial q_6}\right)^{\vee}\right]$$

Convolution

Given $f_1, f_2 \in (L^1 \cap L^2)(SE(3))$

$$(f_1 * f_2)(H) \doteq \int_{SE(3)} f_1(K) f_2(K^{-1}H)dK$$

The Mean on SE(3) Matters

Definition of mean M and covariance Σ on SE(3):

$$\int_{SE(3)} \underbrace{\log(M^{-1}H)}_{1} f(H)dH = \mathbb{O} \quad \text{(3) Mean Def.}$$

$$\Sigma = \int_{SE(3)} \log^{\vee}(M^{-1}H)[\log^{\vee}(M^{-1}H)]^{T} f(H)dH \quad \text{(4) Covariance Def.}$$

Assume $||M^{-1}H - \mathbb{I}|| \ll 1$, then one can perform Taylor expansion on 1

as $\log(\mathbb{I} + X) = X - \frac{1}{2}X^2 + \frac{1}{3}X^3 - ...$ where $X = M^{-1}H$.

Another two ways to approximate the mean can be defined by keeping the 1st order and 2nd order terms.

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How Computing the Mean on SE(3) Matters

New approximations of M based on 1st and 2nd order approximation of the definition of mean

$$\int_{SE(3)} (M^{-1}H-) f(H) dH \approx \mathbb{O}.$$

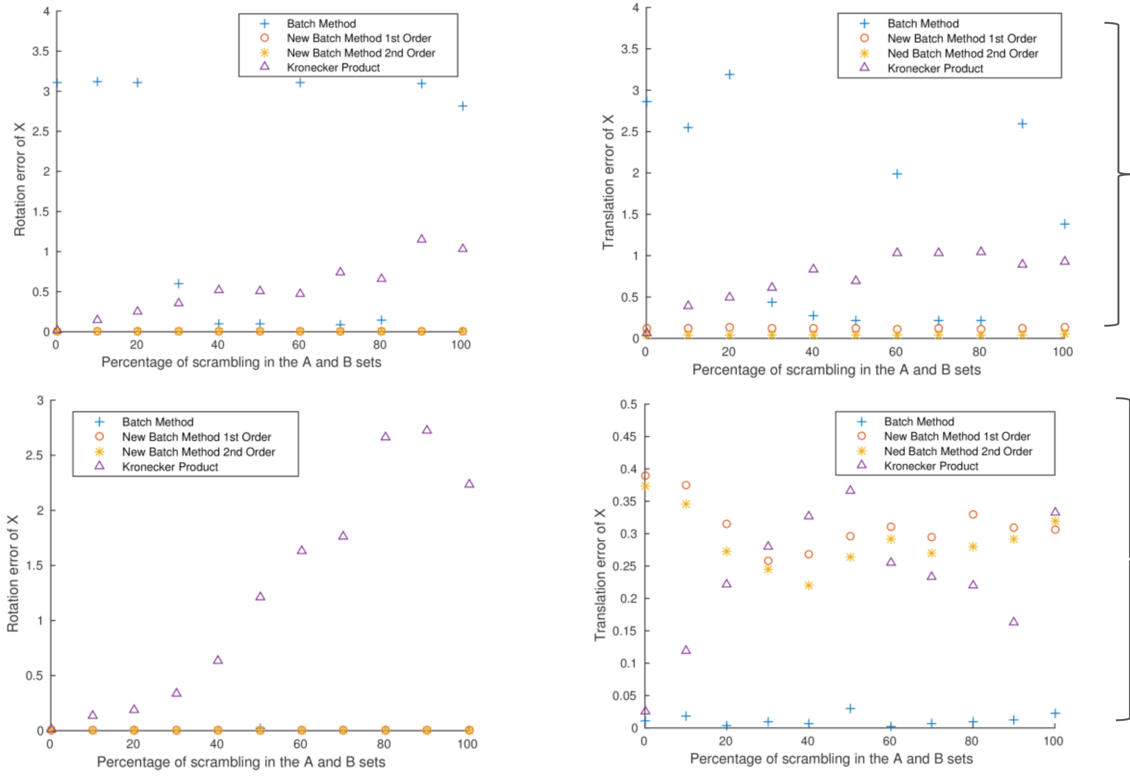
1st order based mean equation

$$\int_{SE(3)} \left(2H - \frac{1}{2} H M^{-1} H - \frac{3}{2} M\right) f(H) dH \approx \mathbb{O}.$$

2nd order based mean equation

Instead of just the approximations of the original mean equation, these two means turned out to be able to characterize certain distributions of the data clouds {A} and {B}, and give X close to the ground truth.

Numerical Comparison



Data cloud with distribution 1

Data cloud with distribution 2

Acknowledgements

This work was supported by NSF Grant RI-Medium: 1162095.

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Thank you for listening

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Simultaneous Hand-Eye and Robot-World Calibration by Solving the AX=YB Problem without Correspondence

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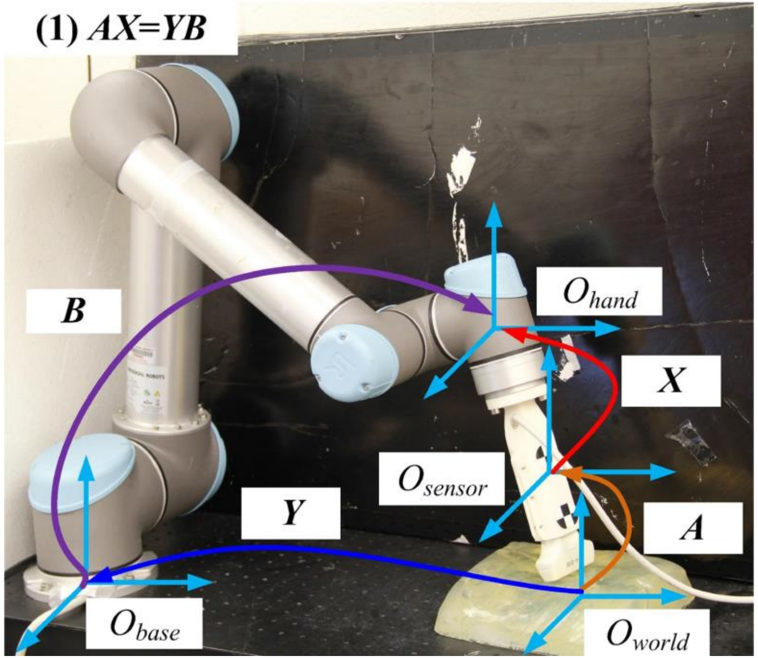
Problem of Interest

- Hand-eye Robot-world calibration
- Most of the algorithms assume exact correspondence which is not always true
- We propose a probabilistic method that can recover the correspondence of shifted data streams and solve for X and Y on its own, and/or augment Other AX=YB solvers.



The AX = YB in a Ultrasound Sensor Calibration Setup

Mathematical Formulation

$$A_i X = Y B_i$$

$$(f_A * \delta_X)(H) = (\delta_Y * f_B)(H)$$

$$\boxed{M_A X = Y M_B}$$

$$\boxed{Ad(X^{-1})\,\Sigma_A\, Ad^T(X^{-1}) = \Sigma_B}$$

(1) Mean Equation

(2) Covariance Equation

Candidates: X_k where k = 1,2,3,4 → ? ? ?

Can't get X as in AX=XB because the similarity transformation between A and B no longer exist in the AX=YB setting

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Screw representation of SE(3)

$$H = \begin{pmatrix} e^{\theta\hat{n}} & (I_3 - e^{\theta\hat{n}})p + dn \\ 0^T & 1 \end{pmatrix}$$

where θ is the angle of rotation, d is the translation along the rotation axis, n is the unit vector representing the axis of rotation and p is the position of a point on the line relative to the origin of a space-fixed reference frame with $p \cdot n = 0$.

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Mathematical Formulation

However, given the candidates of X_k, we can design a method that can recover the correspondence between two shifted data streams $\{A_i\}$ and $\{B_j\}$.

$$AX_k = X_k B^k \quad \text{where} \quad B^k = X_k^{-1} Y_k B$$

$$\Downarrow$$

$$\theta_{A_i} = \theta_{B_i^k}, d_{A_i} = d_{B_i^k}.$$

which come from the Euclidean-group invariants for AX=XB calibration. The invariants can be used to recover the correspondence between shifted data streams.

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which come from the Euclidean-group invariants for AX=XB calibration. The invariants can be used to recover the correspondence between shifted data streams.

Mathematical Formulation

Given two sequences$\{\theta_{A_i}\}$ and $\{\theta_{B_j^k}\}$ corresponding to $\{A_i\}$ and $\{B_j^k\}$

$$\theta_{1,k} = \frac{(\theta_{A_i} - \mu_A)}{\sigma_A}, \theta_{2,k} = \frac{(\theta_{B_j^k} - \mu_{B^k})}{\sigma_{B^k}}$$

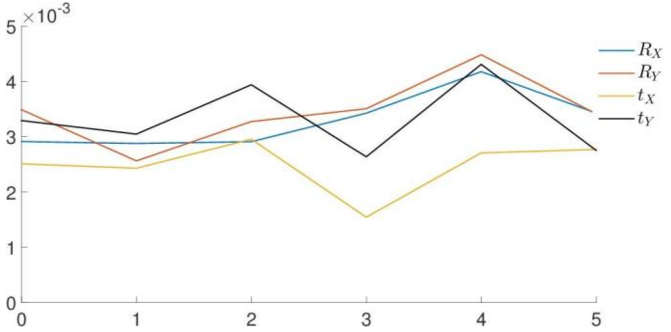
$$Corr(f,g) = f \star g = \mathcal{F}^{-1}[\mathcal{F}(f) \cdot \mathcal{F}^*(g)]$$

By further maximizing the correlation function $\tau_{shift} = \underset{index}{argmax}(Corr(\theta_{1,k}, \theta_{2,k}))$, one is able to recover the amount of shift between two data streams. The optimal (X, Y) pair can be picked out as follows.

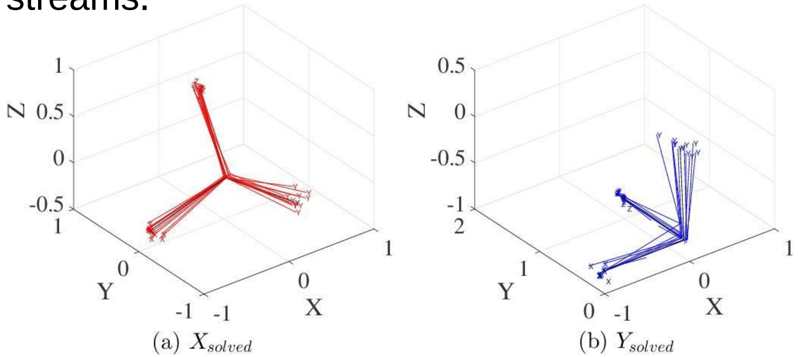
$$(X,Y) = \underset{(X_k,Y_k)}{argmin} \frac{1}{n} \sum_{i=1}^{n} (\| \theta_{A_i} - \theta_{B_i^k} \| + \| d_{A_i} - d_{B_i^k} \|)$$

Numerical Comparison

The probabilistic method dealing with shifted data streams.



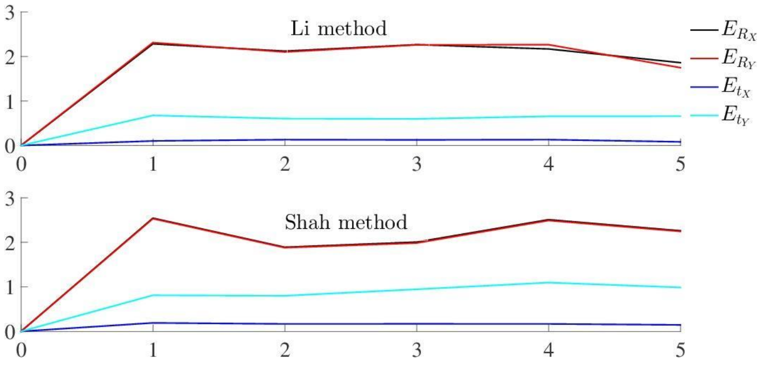
The translational and rotational errors versus the shift between data streams {Ai} and {Bi}

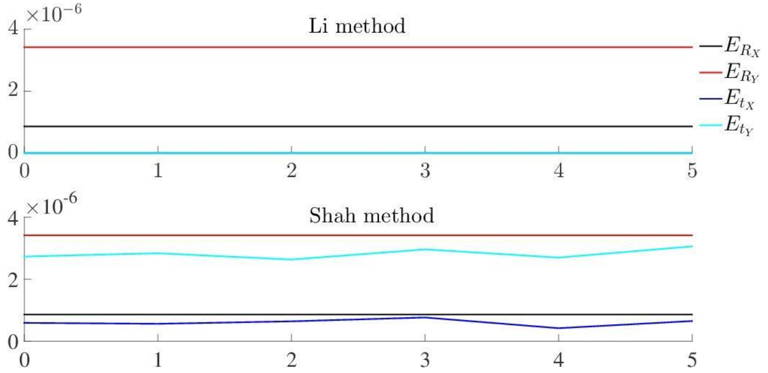


(a) The solved X (in red) and the actual X (in black) for 10 simulation trials with covariance noise of 0.05 and shift of 2.
(b) The solved X (in blue) and the actual Y (in black) for 10 simulation trials with covariance noise of 0.05 and shift of 2.

Numerical Comparison

Li and Shah before and after correspondence recovery by our probabilistic approach





Orientation and translation errors of X and Y versus shift using Li's and Shah's methods without recovering correspondence

Orientation and translation errors of X and Y versus shift using Li's and Shah's methods after recovering correspondence

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