Probabilistic Approaches to the AXB = YCZ Calibration Problem in Multi-Robot Systems

Qianli Ma, Zachariah Goh, Gregory S. Chirikjian
Department of Mechanical Engineering,
Johns Hopkins University, Baltimore, MD, USA

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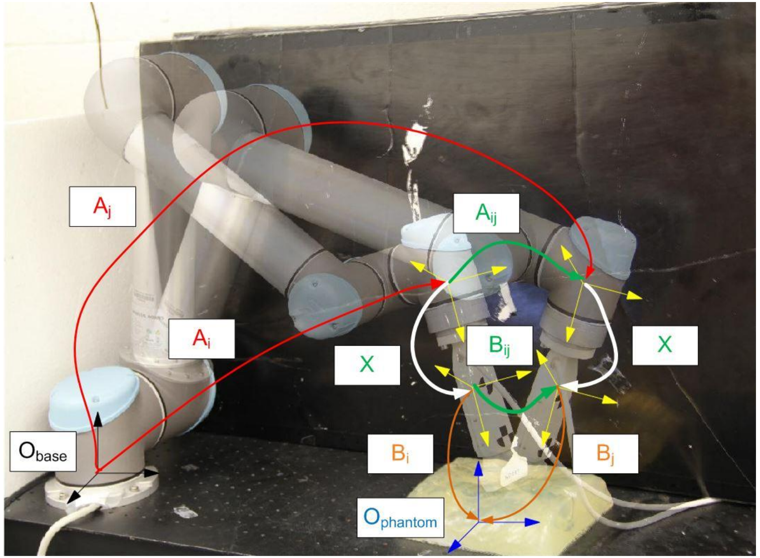
The AX = XB and AX = YB Calibrations



AX = XB Robot Hand-Eye Calibration

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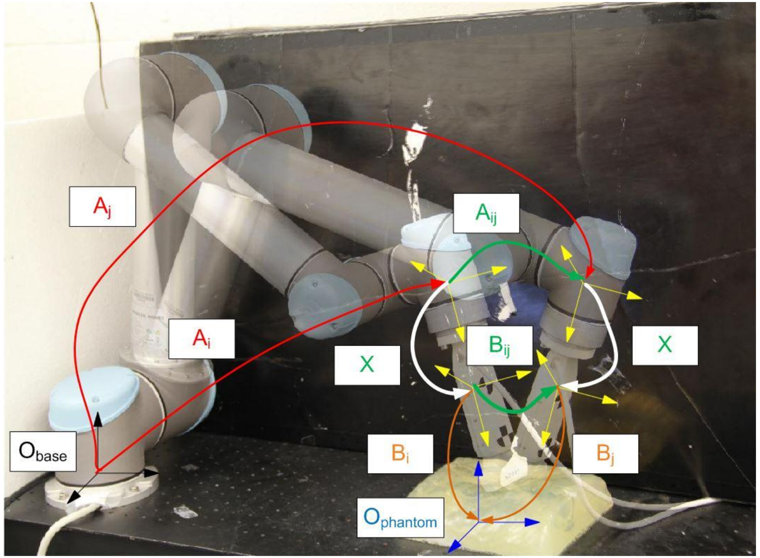
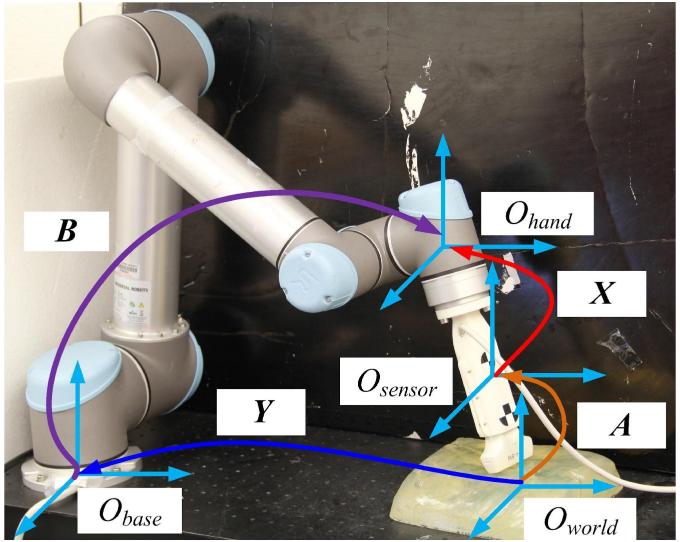
The AX = XB and AX = YB Calibrations



AX = XB Robot Hand-Eye Calibration



AX = YB Hand-Eye and Robot-World Calibration

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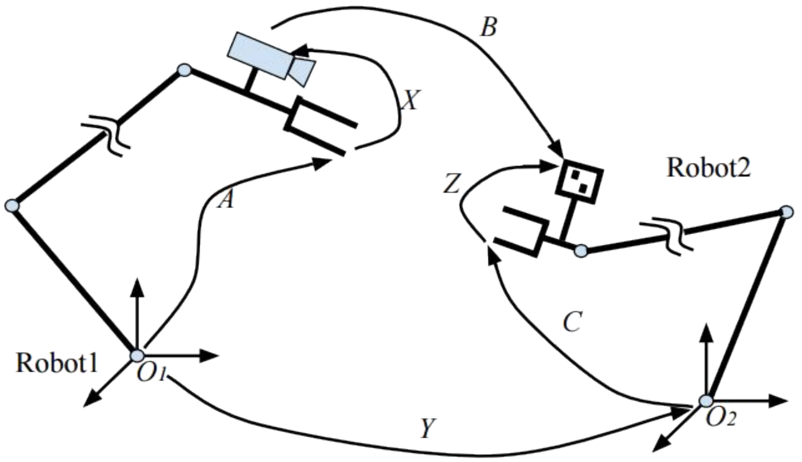
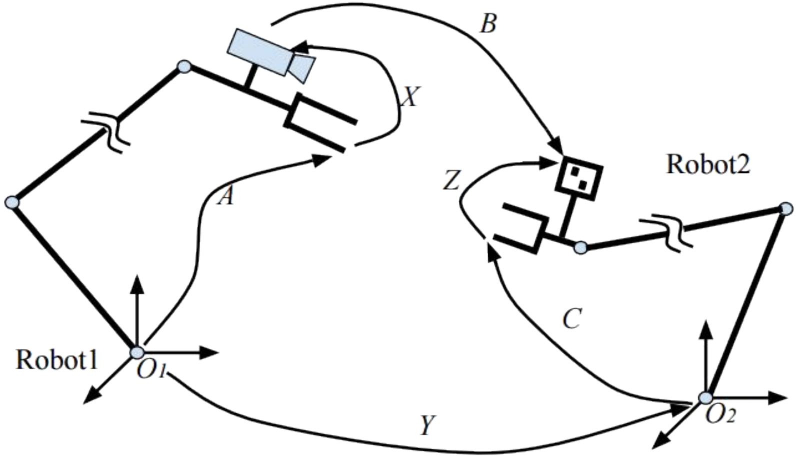
The AXB = YCZ Calibration



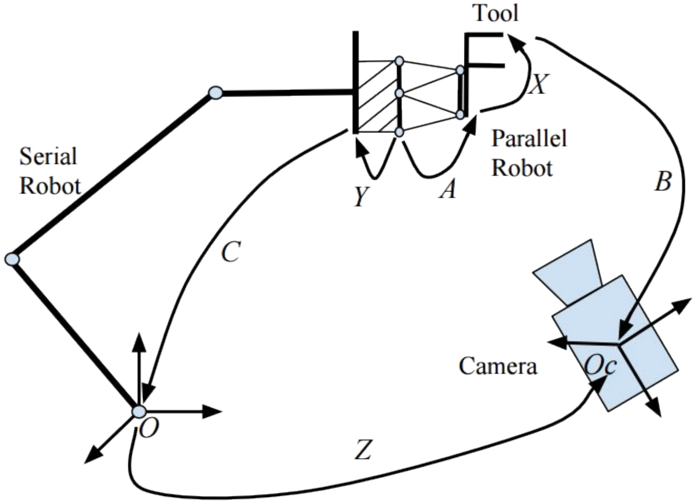
Hand-Eye (X), Robot-Robot (Y) and Tool-Flange (Z) Calibrations of a Dual Arm System[1,2]

[1] Wang, Jiaole, et al. IROS 2014. [2] Wu, Liao, et al. T-RO 2016.

The AXB = YCZ Calibration



Hand-Eye (X), Robot-Robot (Y) and Tool-Flange (Z) Calibrations of a Dual Arm System[1,2]

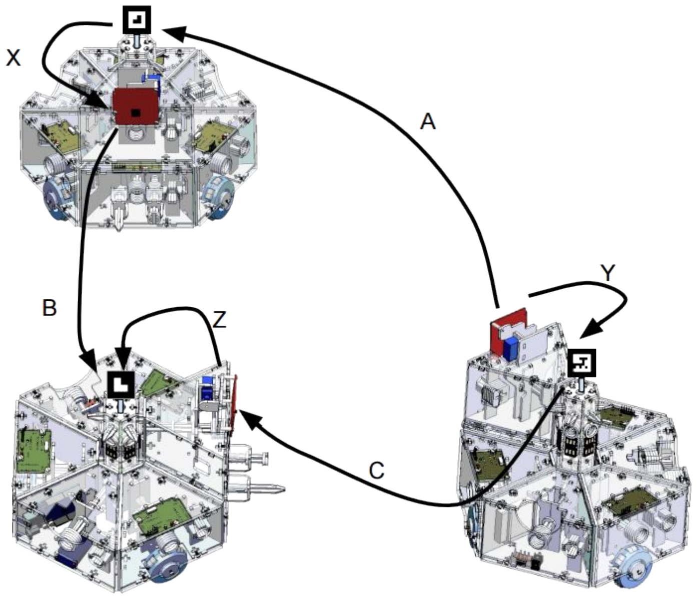


Flange-Base (Y), Camera-Base (Z) and Tool-Gripper (X) Calibrations of a Serial-Parallel Manipulator[3]

[1] Wang, Jiaole, et al. IROS 2014. [2] Wu, Liao, et al. T-RO 2016.

[3] Yan, S.J., et al. Robotica 2015.

The AXB = YCZ Calibration



Triple Hand-Eye (X, Y, Z) Calibrations of a Multi-Robot System

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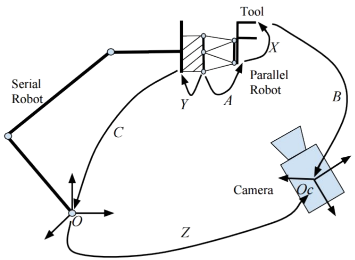
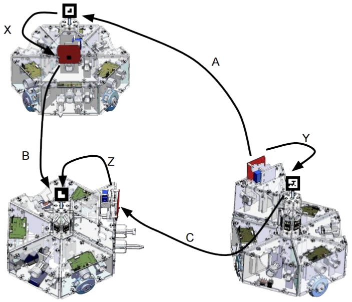
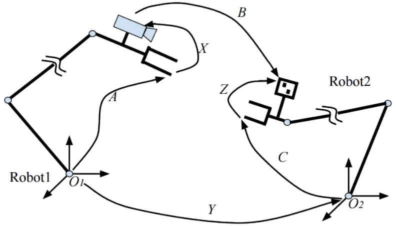
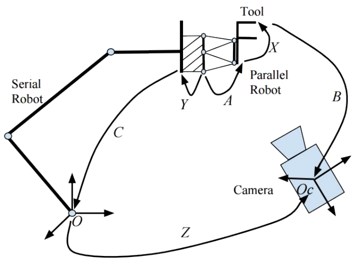
Mathematical Background

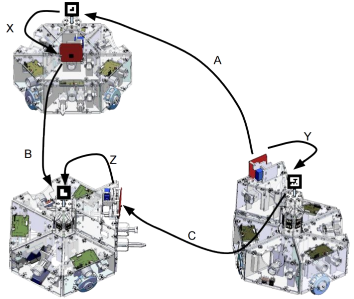


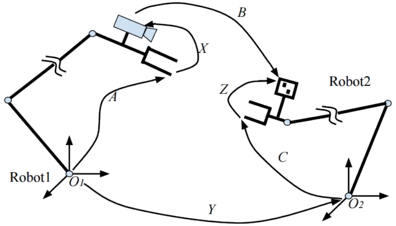




Mathematical Background



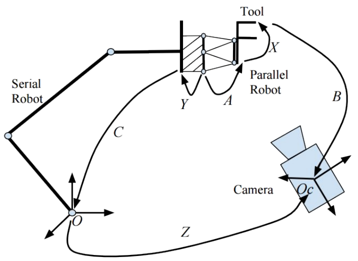


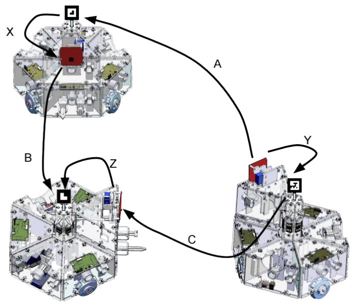


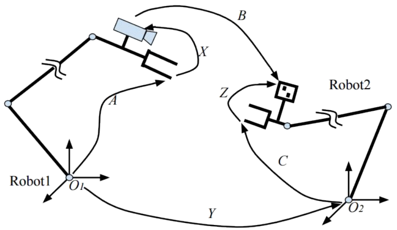
$$A = \{A_i\} \rightarrow f_A(H) \quad \text{with mean } M \text{ and covariance } \Sigma$$

$$X \rightarrow \delta_X(H)$$

Mathematical Background







$$A = \{A_i\} \rightarrow f_A(H) \text{ with mean } M \text{ and covariance } \Sigma$$

$$X \rightarrow \delta_X(H)$$

$$\int_{SE(3)} \log(M^{-1}H) f(H) dH = \mathbb{O}$$ Mean Def.

$$\Sigma = \int_{SE(3)} \log^{\vee}(M^{-1}H)[\log^{\vee}(M^{-1}H)]^T f(H) dH$$ Covariance Def.

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Mathematical Formulation

$$A_i X B_i = Y C_i Z$$

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Mathematical Formulation

$$A_i X B_i = Y C_i Z$$

$$\Downarrow$$

$$(f_A * \delta_X * f_B)(H) \approx (\delta_Y * f_C * \delta_Z)(H).$$

Independent of correspondence

Mathematical Formulation

$$A_i X B_i = Y C_i Z$$

$$(f_A * \delta_X * f_B)(H) \approx (\delta_Y * f_C * \delta_Z)(H).$$

Independent of correspondence

$$M_A X M_B \approx Y M_C Z$$

(1) Mean Equation on $SE(3)$

$$Ad(B^{-1})Ad(X^{-1})\Sigma_A Ad^T(X^{-1})Ad^T(B^{-1}) + \Sigma_B \approx Ad(Z^{-1})\Sigma_C Ad^T(Z^{-1})$$

(2) Covariance Equation on $\mathbb{R}^{6\times 6}$

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Mathematical Formulation

$$A_i X B_i = Y C_i Z$$

$$(f_A * \delta_X * f_B)(H) \approx (\delta_Y * f_C * \delta_Z)(H).$$

Independent of correspondence

$$M_A X M_B \approx Y M_C Z$$

(1) Mean Equation on $SE(3)$

$$Ad(B^{-1})Ad(X^{-1})\Sigma_A Ad^T(X^{-1})Ad^T(B^{-1}) + \Sigma_B \approx Ad(Z^{-1})\Sigma_C Ad^T(Z^{-1})$$

(2) Covariance Equation on $\mathbb{R}^{6\times 6}$

Lost Y in the covariance equation

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Case Study with Completely Scrambled Data

$$AXB = YCZ \quad (1)$$

$$C^{-1}Y^{-1}A = ZB^{-1}X^{-1} \quad (2)$$

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Case Study with Completely Scrambled Data

$$AXB = YCZ \quad (1)$$

Covariance Eq. (1) | Mean Eq. (1)

$$C^{-1}Y^{-1}A = ZB^{-1}X^{-1} \quad (2)$$

Mean Eq. (2) | Covariance Eq. (2)

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Case Study with Completely Scrambled Data

$$AXB = YCZ \quad (1)$$

$$C^{-1}Y^{-1}A = ZB^{-1}X^{-1} \quad (2)$$

Covariance Eq. (1) | Mean Eq. (1)

$\{Z_k\}$

Mean Eq. (2) | Covariance Eq. (2)

$\{X_i\}$

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Case Study with Completely Scrambled Data

$$AXB = YCZ \quad (1)$$

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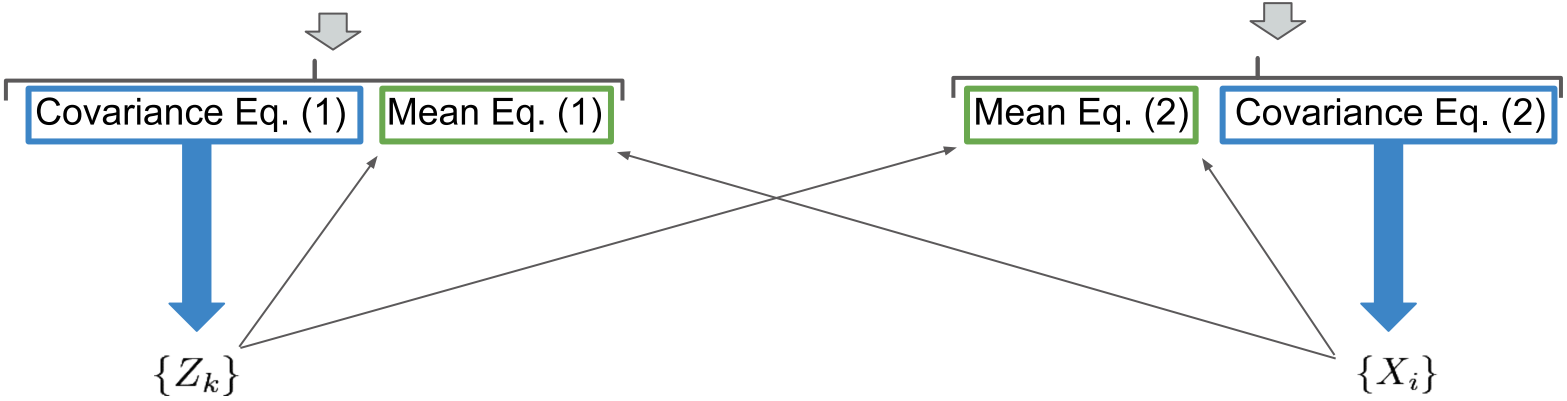

Case Study with Completely Scrambled Data

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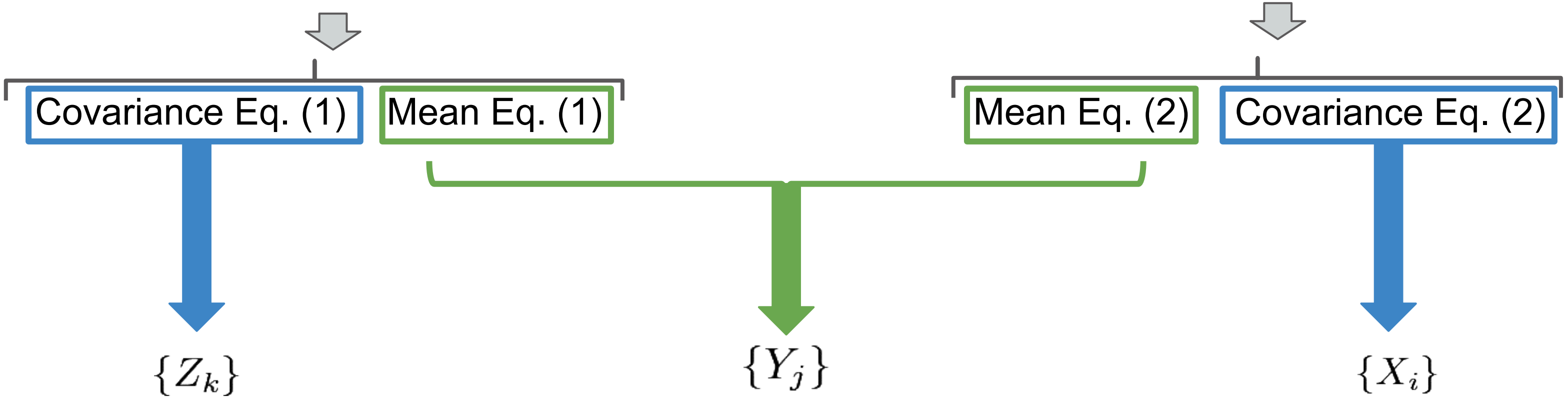

Case Study with Completely Scrambled Data

$$AXB = YCZ \quad (1)$$

$$C^{-1}Y^{-1}A = ZB^{-1}X^{-1} \quad (2)$$

Covariance Eq. (1) → $\{Z_k\}$

Mean Eq. (1), Mean Eq. (2) → $\{Y_j\}$

Covariance Eq. (2) → $\{X_i\}$

$$\min_{i,j,k} f(X_i, Y_j, Z_k)$$

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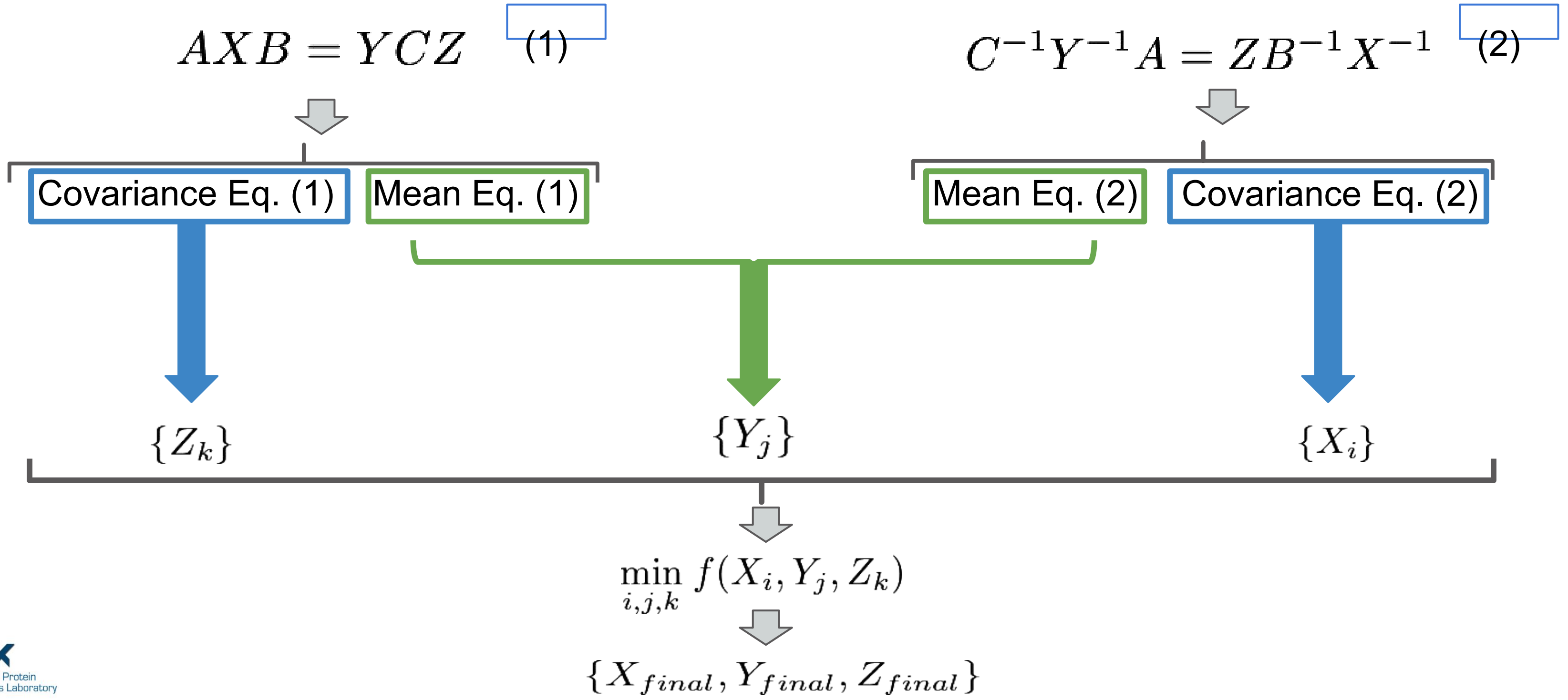
Case Study with Completely Scrambled Data



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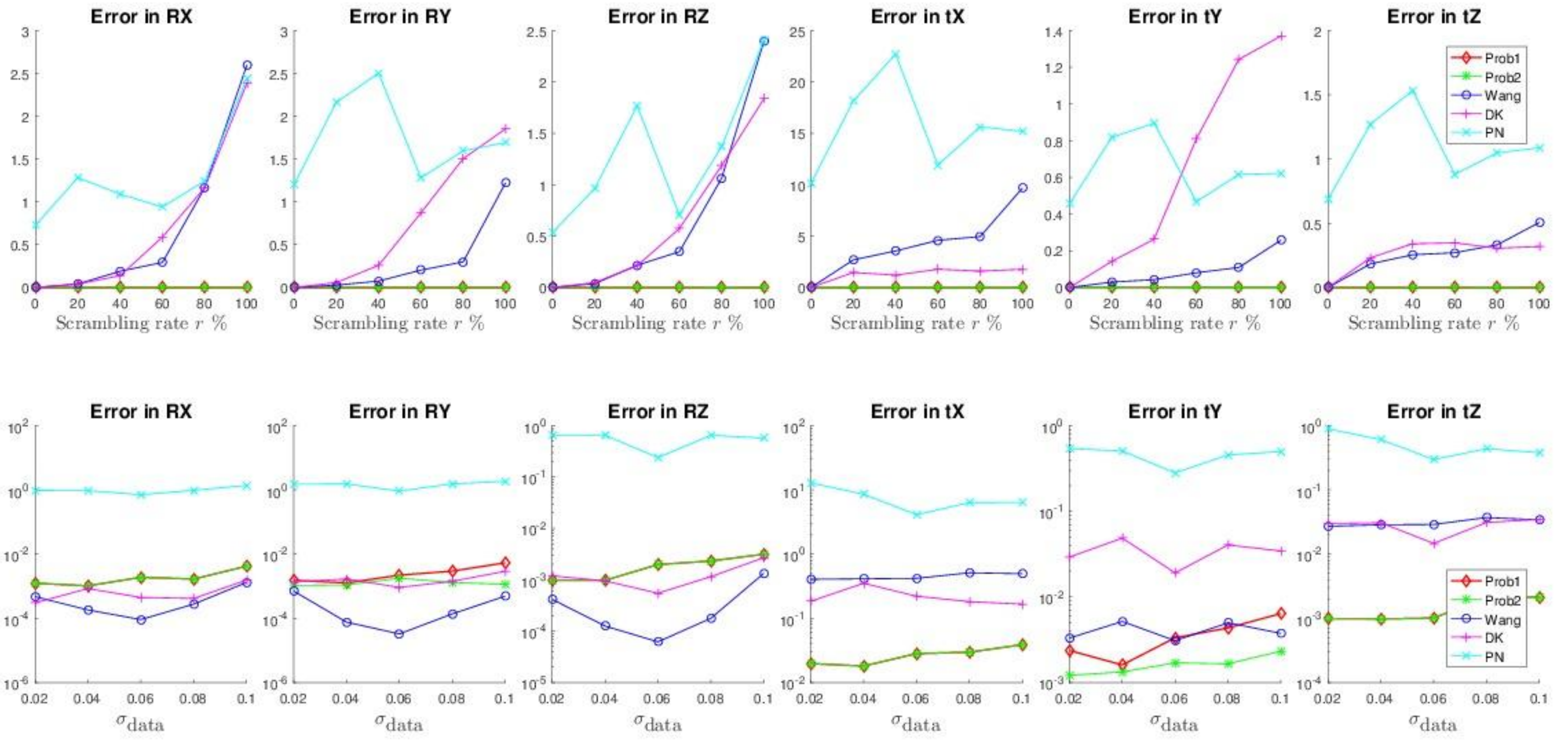
Comparison between Probabilistic Methods and Traditionals Methods



Prob1 and *Prob2* are probabilistic methods; *Wang*, *DK* and *PN* are traditional methods

Acknowledgements

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Lab website : https://rpk.lcsr.jhu.edu/

Contact: mqianli1@jhu.edu

Thank you for listening!

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